Some Practical Aspects of Model Predictive Control

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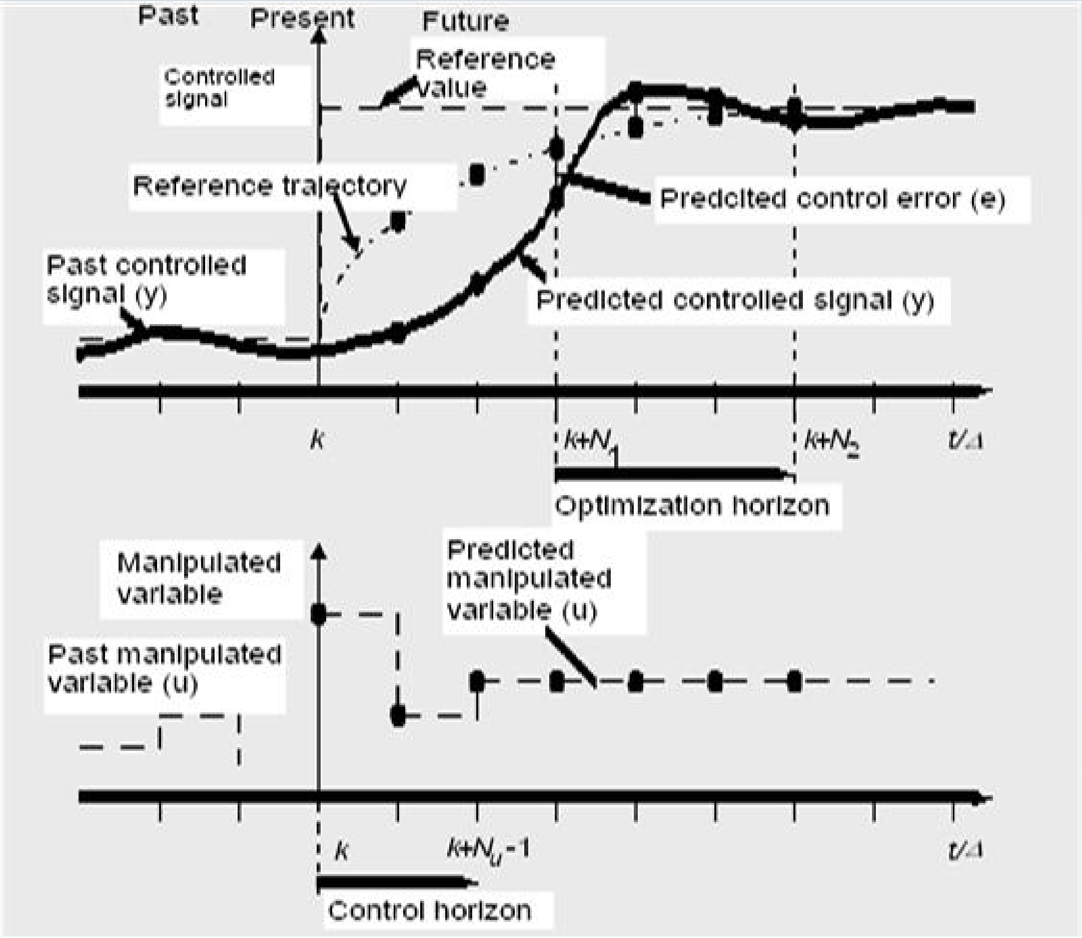
Outline

- Introduction
- Two techniques for saving computational effort
  - Coincidence points
  - Blocking technique
- IMC representation of GPC
- Handling of constraints

Main features of predictive control



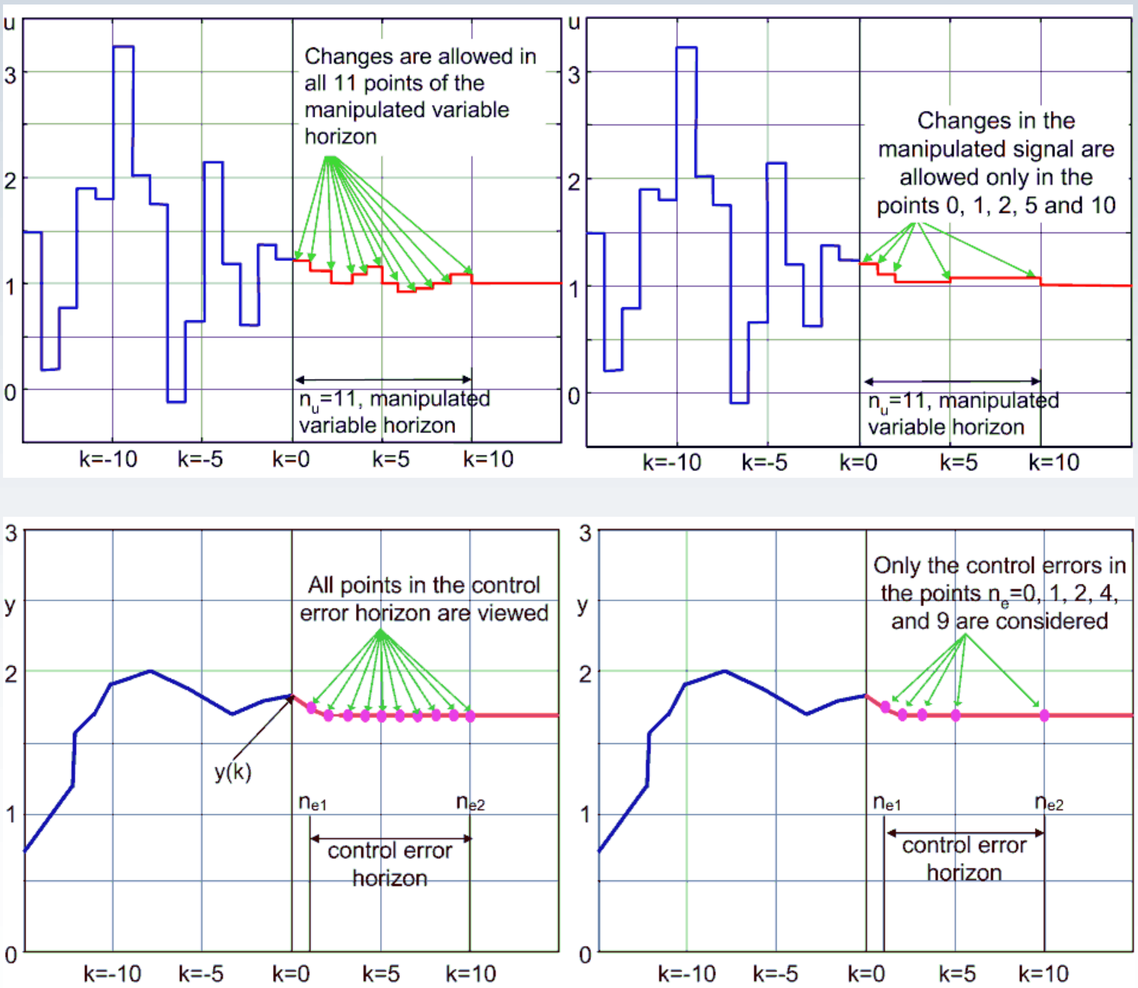
The cost function:

$$J = \sum_{n_e = n_{e1}}^{n_{e2}} \left[y_{ref}(k+n_e+d) - \hat{y}(k+n_e+d) \right]^2 + \lambda_u \sum_{j=1}^{n_u} \Delta u(k+j-1)^2 \Rightarrow \min_{\Delta u}$$

Optimisation under constraints can be a time consuming operation especially with long prediction horizons.

Real time applications require techniques reducing the computation time significantly.

Saving computational effort using coincidence points and blocking technique



How to allocate the points in the horizons?

The number of the considered points can be reduced even by a factor 10.

A two-stage optimisation process has been executed:

1. The length of the prediction and the control horizons were optimised without reducing the computation points in the horizons.
2. The allocation of the computation points in both horizons were optimised; the aim was to approximate the control performance without the reduction of the points.

Choice of the horizons

- Sample second order aperiodic processes were considered (Tsmall and Tbig between 1 and 30 sec.). The sampling time was 0.2sec.
- The length of the horizons was optimised according to 3 criteria:

  the settling time

  the overshoot

  the length of the horizons, only if

$$n_{e2,\max} > 4T_\Sigma$$
$$n_{u,\max} > T_\Sigma$$

Optimisation by a genetic algorithm (Sekaj).

$$J = J_{set} + J_{ov} + J_h = \lambda_{set} t_{98\%} + \lambda_{ov} \left| y_{\max} - y_{ref} \right| + \lambda_{e,hor} V_{ne} + \lambda_{u,hor} V_{nu} \Rightarrow \min_{n_{e1}, n_{e2}, n_u, \lambda_u}$$

with $V_{ne} = \sum_{i=n_{e1}}^{n_{e2}} \left\{ \begin{array}{lll} 0 & \text{for} & d+1+i \le 4\dfrac{T_\Sigma}{\Delta T} \\ i & \text{for} & d+1+i > 4\dfrac{T_\Sigma}{\Delta T} \end{array} \right\}$ and $V_{nu} = \sum_{i=1}^{n_u} \left\{ \begin{array}{lll} 0 & \text{for} & i-1 \le \dfrac{T_\Sigma}{\Delta T} \\ i & \text{for} & i-1 > \dfrac{T_\Sigma}{\Delta T} \end{array} \right\}$

Rules of thumb for the choice of the horizons

- The beginning of the prediction horizon:

$$n_{e_1} \approx 0.04\left(\frac{T_{big}}{\Delta T} - \frac{T_{small}}{\Delta T}\right)$$

- The end of the prediction horizon:

$$n_{e_2} \approx \begin{cases} 18\dfrac{T_{big}}{T_{small}}, & \text{for } \dfrac{T_{big}}{T_{small}} \geq 10 \\ \dfrac{1}{3}\left(\dfrac{T_{big}}{\Delta T} + \dfrac{T_{small}}{\Delta T} + 150\right), & \text{for } \dfrac{T_{big}}{T_{small}} < 10 \end{cases}$$

- The control horizon:

$$n_u \approx \begin{cases} 0.045\dfrac{T_{big} + T_{small}}{\Delta T}, & \text{for } \dfrac{T_{big}}{T_{small}} \geq 10 \\ 0.0554\dfrac{T_{big} + T_{small}}{\Delta T}, & \text{for } \dfrac{T_{big}}{T_{small}} < 10 \end{cases}$$

Reducing the number of points within the horizons

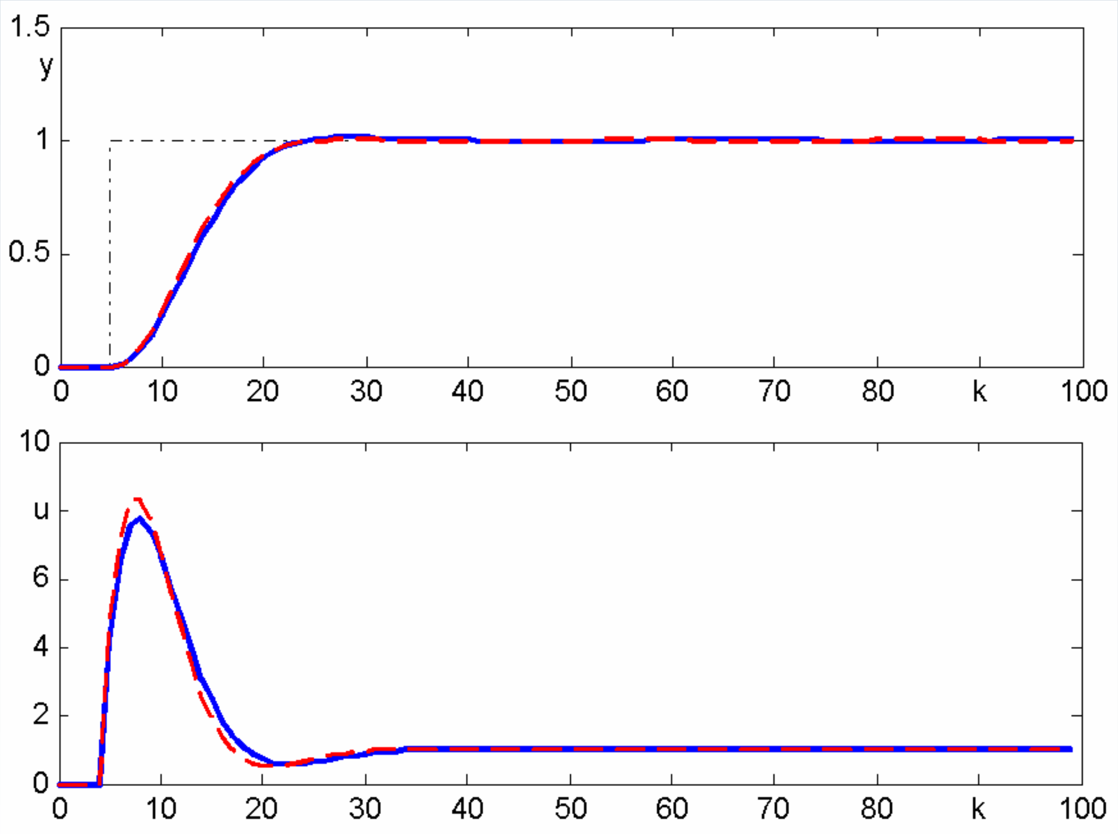
- The next task was to reduce the number of points to
  Pe=20 and Pu=10.
- The aim of optimisation was to get a close performance to the control with optimised horizons with full number of points.
- The cost function:

$$J = J_e + J_u + J_h = \lambda_{e,dev} \sum_{k=0}^{200} \left(y_{full}(k) - y_{red}(k)\right)^2 + \lambda_{u,dev} \sum_{k=0}^{200} \left(u_{full}(k) - u_{red}(k)\right)^2$$

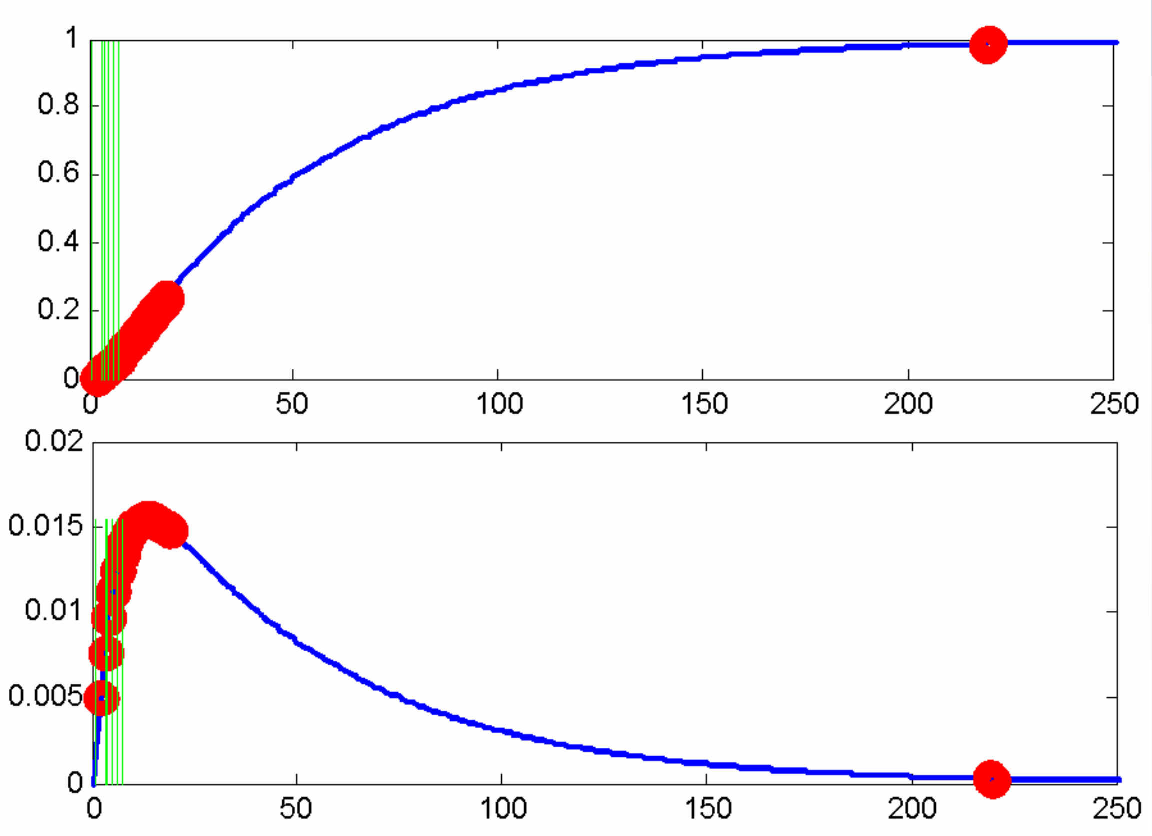
$$+ \lambda_{Pe} V_{Pe} + \lambda_{Pu} V_{Pu} \Rightarrow \min_{P_e, P_u}$$

$$\text{with} \quad V_{Pe} = \sum_{i=1}^{N_{B,n_e}} \begin{cases} 0 & \text{for } P_i \le 4T_\Sigma \\ i & \text{for } P_i > 4T_\Sigma \end{cases} \quad \text{and} \quad V_{Pu} = \sum_{i=1}^{N_{B,nu}} \begin{cases} 0 & \text{for } P_i \le T_\Sigma \\ i & \text{for } P_i > T_\Sigma \end{cases}$$

Step and impulse responses with full and reduced number of points



Optimal allocation of the coincidence points



Polynomial forms of unconstrained MPC GPC

- The cost function:

$$J = \sum_{j=N_1}^{N_2} \delta(j)\left[\hat{y}(t+j \mid t) - r(t+j)\right]^2 + \sum_{j=1}^{N_u} \lambda_u(j)\left[\Delta u(t+j-1)\right]^2$$

- The ARIMAX model of the process:

$$A(q^{-1})y(t) = B(q^{-1})u(t-1) + T(q^{-1})\frac{e(t)}{\Delta}$$

- The control increment:

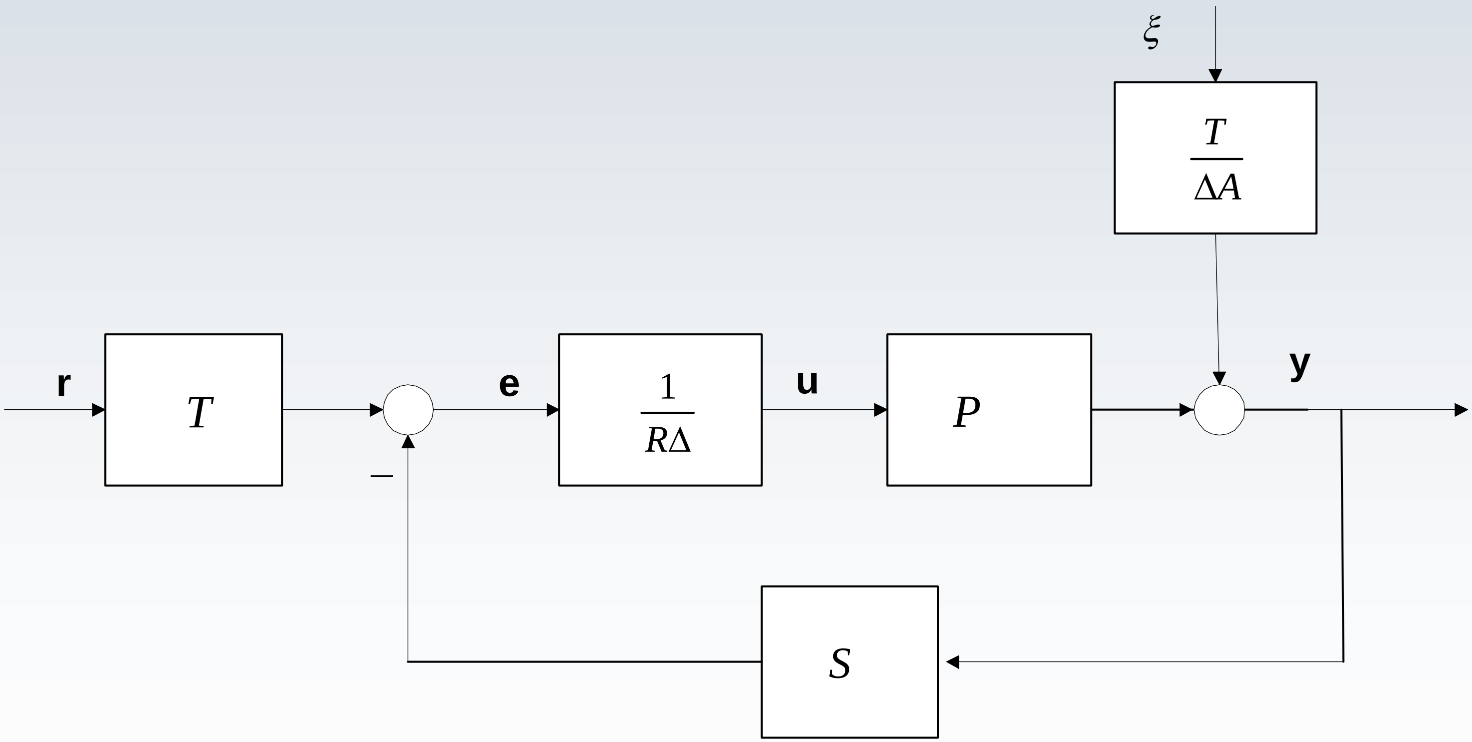
$$\Delta \mathbf{u} = \left[\mathbf{H}_\Delta^T \mathbf{H}_\Delta + \lambda_u \mathbf{I}\right]^{-1} \mathbf{H}_\Delta^T \left(\mathbf{y}_r - \hat{\mathbf{y}}_{free}\right)$$

receding horizon strategy

$$\Delta u(t) = \mathbf{K}(\mathbf{y}_r - \hat{\mathbf{y}}_{free}) = \sum_{i=N_1}^{N_2} k_i \left[y_r(t+i) - y_{free}(t+i) \right]$$

GPC and its polynomial RST structure

- In unconstrained case the GPC has an equivalent two-degree-of-freedom polynomial structure, the RST structure



GPC and its polynomial RST structure

- Advantages of RST structure:
  - easy and cheap implementation (low cost solution)
  - Quick, since there is no optimization in each cycle

The R, S and T polynomials can be computed using Diophantine equations

$$T(q^{-1}) = E_j(q^{-1})\Delta A(q^{-1}) + q^{-j} F_j(q^{-1})$$
$$E_j(q^{-1}) B(q^{-1}) = H_j(q^{-1}) T(q^{-1}) + q^{-j} I_j(q^{-1})$$

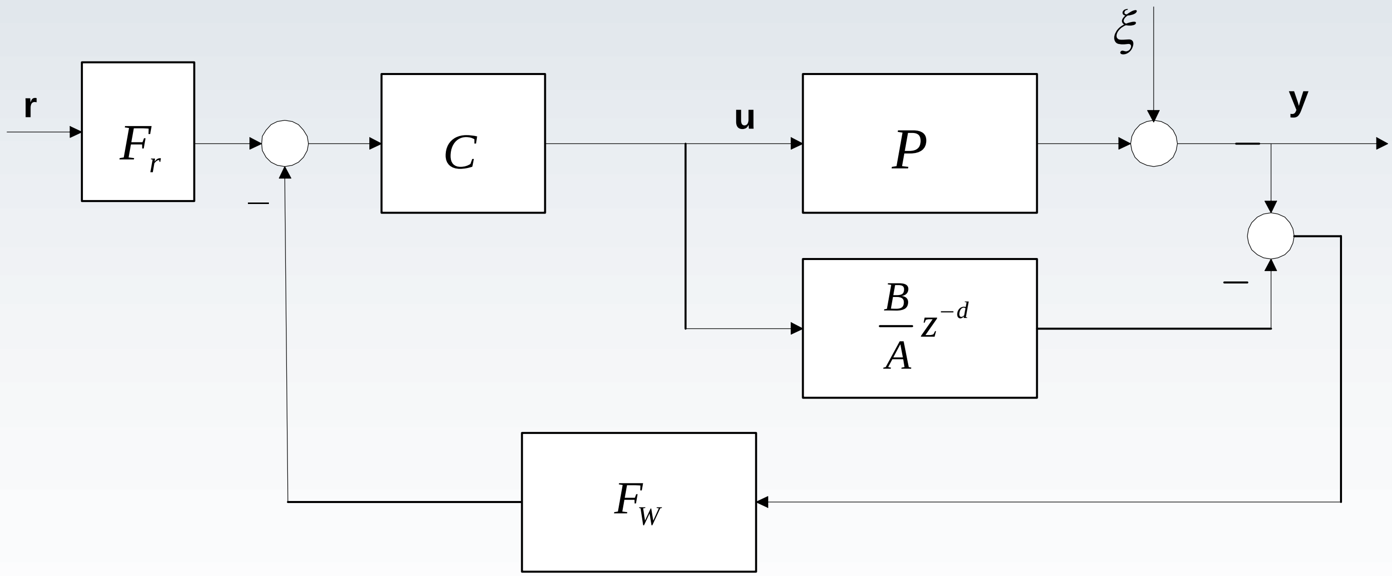
- Based on the GPC parameters, the polynomials will be:

$$R(q^{-1}) = \frac{T(q^{-1}) + q^{-1}\sum_{i=N_1}^{N_2} k_i I_i}{\sum_{i=N_1}^{N_2} k_i} \qquad S(q^{-1}) = \frac{\sum_{i=N_1}^{N_2} k_i F_i}{\sum_{i=N_1}^{N_2} k_i}$$

$$T(q^{-1}) = \text{free parameter}$$

Equivalent IMC structure of GPC

- Define the IMC-based structure of the GPC (starting from the RST structure)



Equivalent IMC structure of GPC

- The elements of the control system are:

$$C = \frac{SA}{F_w(R\Delta A + SBq^{-d})}, \qquad \frac{T(q^{-1})}{S(q^{-1})} = \frac{F_r(q^{-1})}{F_W(q^{-1})}, \qquad F_r = T \quad F_w = S$$

- In this form it can be used only for stable process

- The values of R, S, T, respectively F_r, F_w, C are varying in function of the GPC parameters: N_1, N_2 (prediction horizon), N_u (control horizon), λ weighting factor

Effect of parameter changes
Example

The plant:

$$P(s)=\frac{1}{(1+0.67s)(1+0.33s)}$$

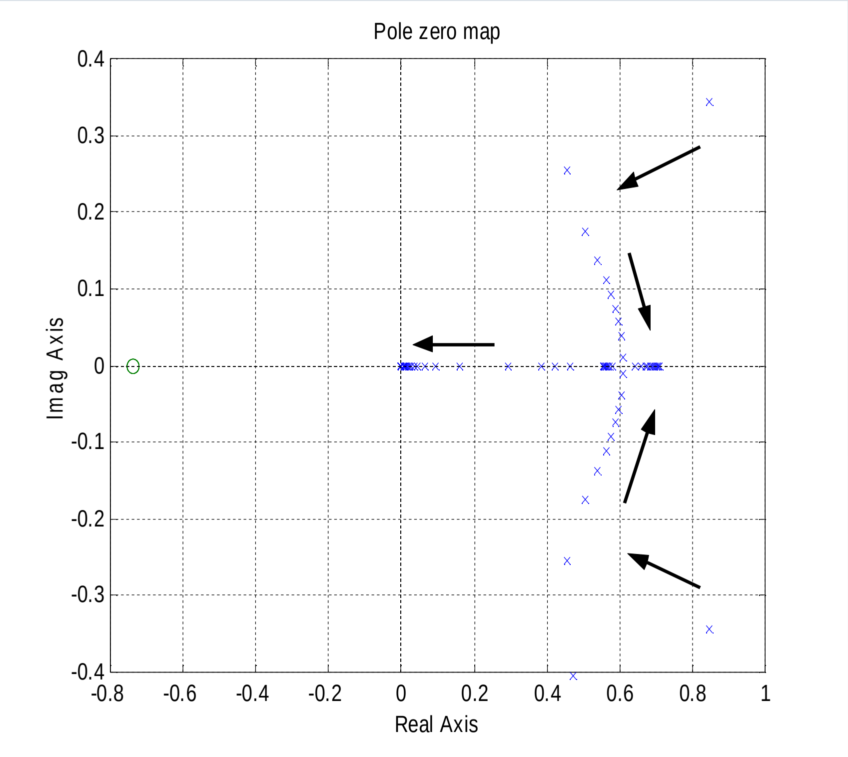
Sampling time: h=0.2

$$P(z)=\frac{0.0674z^{-1}+0.0499z^{-2}}{1-1.2874z^{-1}+0.4047z^{-2}}$$

Tuning parameters:

$$N_1=1,\ N_u=1,\ d=0,\ \lambda_u=0.1,\ \lambda_y=1$$

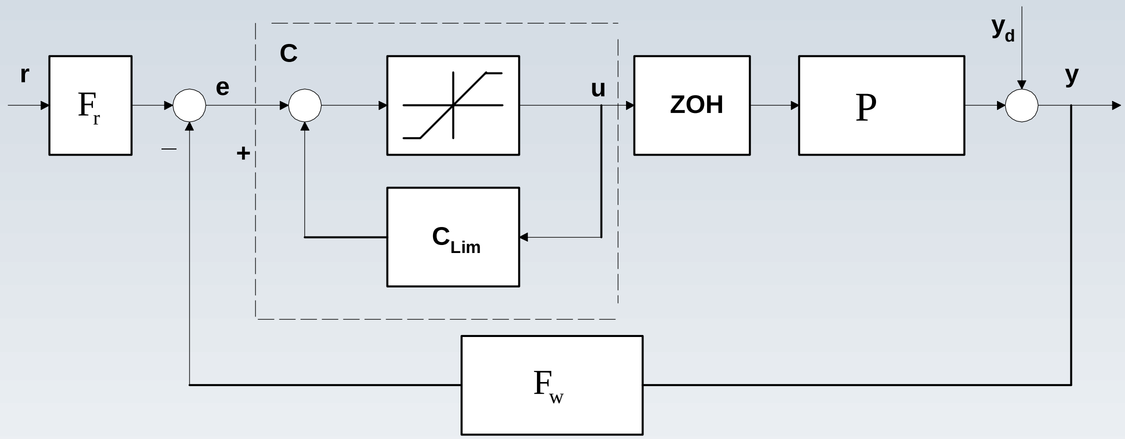
$$N_2=(1:1:25)$$

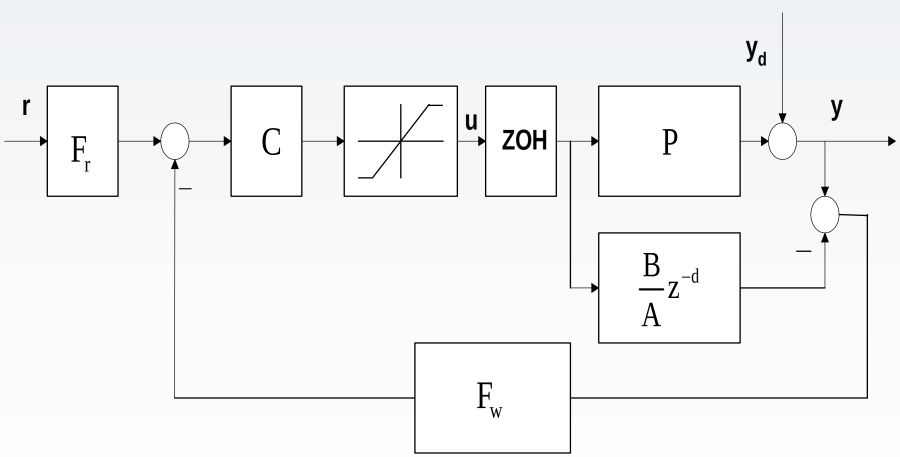


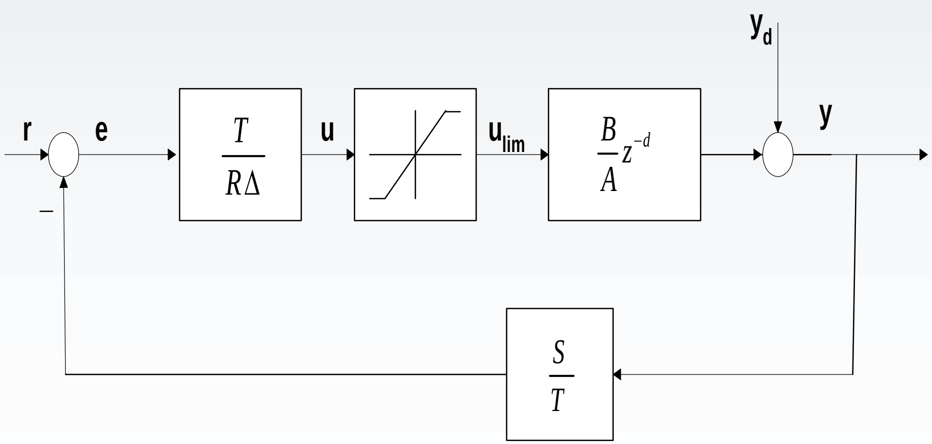
Choice of polynomial T influences robustness.

Constraint handling in IMC structure of GPC

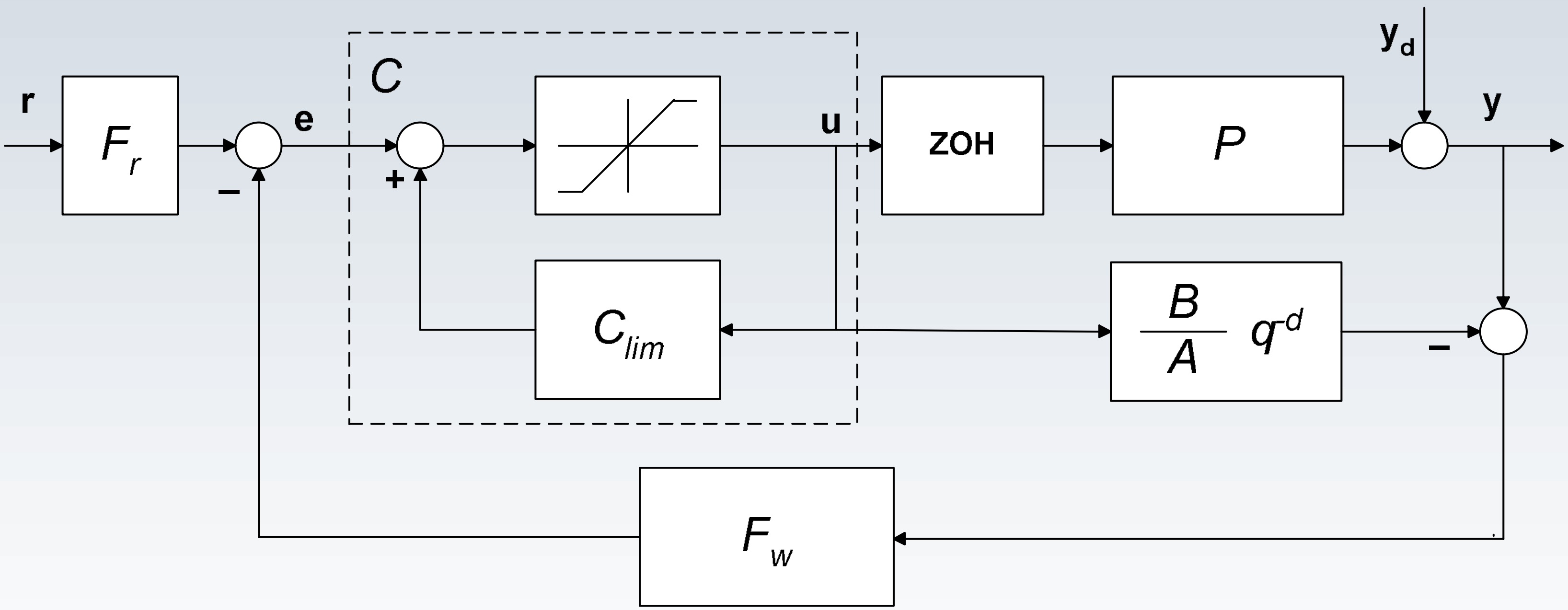
Constraint handling in GPC: the IMC structure is favorable







IMC with feedback saturation



$$C_{Lim}(q^{-1}) = \frac{C(q^{-1}) - 1}{C(q^{-1})}$$

Example

- Case of a first order system: $P(s) = \dfrac{1}{1+s}$
- Sampling time 0.1.
- GPC parameters: $N_1 = 1 \quad N_2 = 3$

  $N_u = 1 \quad \lambda_u = 0$
- The RST parameters: $R = 0.2037$

  $S = 2.9366 - 1.9366\, z^{-1}$

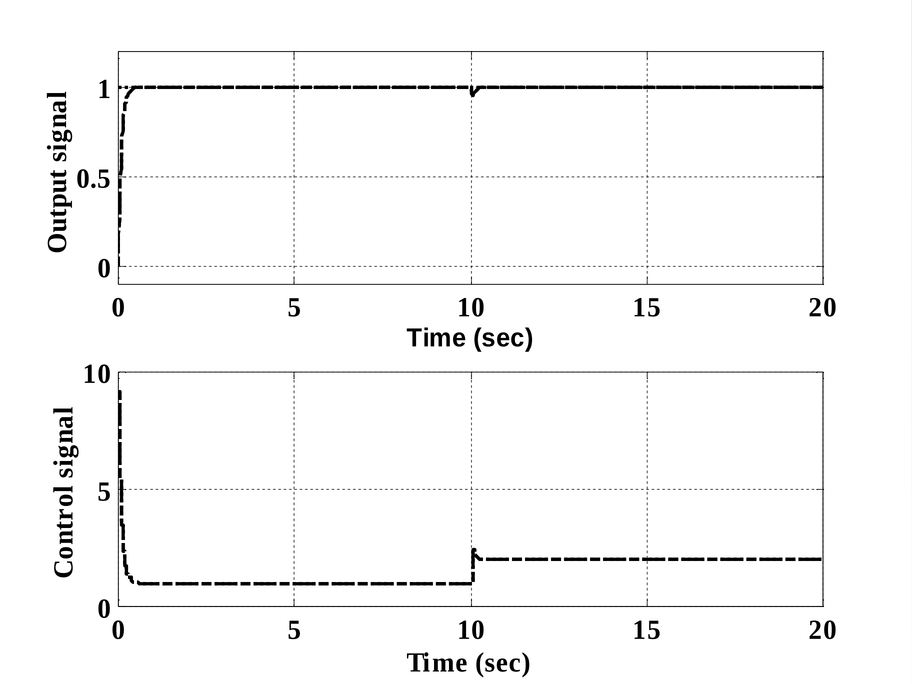
  $T = 1$
- The IMC parameters: $F_r = 1$

  $F_W = 2.9366 - 1.9366\, z^{-1}$

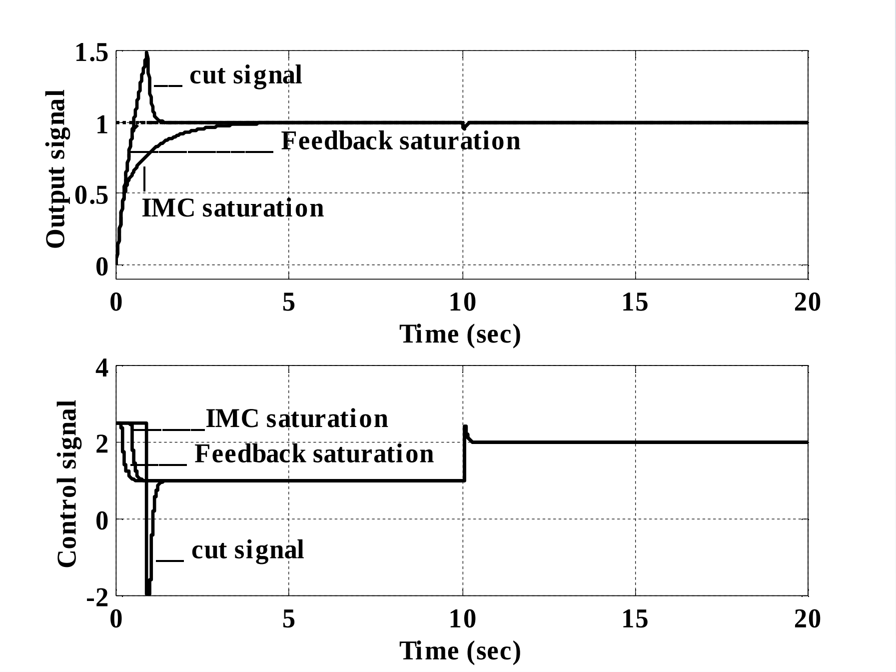
$$C_{Lim}(z) = \frac{0.7963z - 0.7963}{z - 0.9048} \qquad C = \frac{4.9098 - 4.443\, z^{-1}}{1 - 0.5328\, z^{-1}}$$

Examples

Unconstrained case:



Constrained case:



Summary

- Computation time in predictive control algorithms can be reduced drastically using coincidence points and blocking technique.
- Rules for optimal choice of the points are given.
- With parametric forms of GPC the calculations are also reduced.
- IMC equivalent of GPC structure is given.
- Aposteriori constraint handling is needed.
- The best solution is to put the dynamics in an inner feedback of the static saturation element.

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