D-decomposition Revisited

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Introduction

- Design of low-order controllers
- Robustness analyses

$$\min_{K \in \partial D_n} |K|$$

$$\dot{x} = Ax + Bu$$
$$y = Cx,\ u = Ky$$

$A \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times r}$,
$C \in \mathbf{R}^{m\times n}$, $K \in \mathbf{R}^{r\times m}$

$K \in \Delta$ - given class:

- $K = k \in \mathbf{R}^n$ or $K = k^T$ – single-input or single-output system (r=1 or m=1)
- $K = kI$, $k \in \mathbf{R}$ or $k \in \mathbf{C}$ – system with scalar gain (m=r)
- $K \in \mathbf{R}^{2\times 2}$ – double-input double-output system

Eigenvalue Invariant Region (EIR):

$D_i = \{K \in \Delta$: A+BKC has i stable eigenvalues$\}$

D_n – stability domain

D-decomposition technique

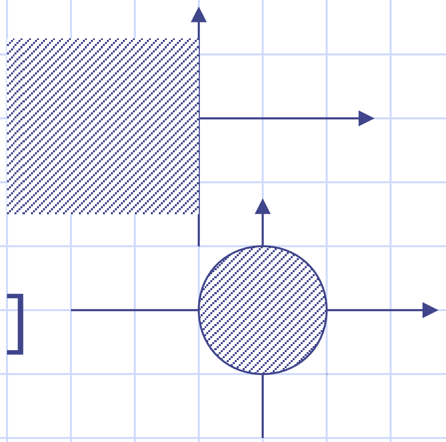
$M(s)= C(I-sA)^{-1}B$ – matrix transfer function (i.e. $y=Mu$)

Eigenvalue Plane		Parameter Space
eigenvalue placement	$\det(I+M(\xi(\omega))K)=0$ ⇒	eigenvalue invariant regions boundary

$\xi(\omega)$ eigenvalue placement boundary

[$\xi(\omega)=j\omega$, $\omega\in[-\infty,+\infty]$ – LHP;

$\xi(\omega)=e^{j\omega}$, $\omega\in[0,2\pi)$ – unit disk]



$K(\omega)$

Note: $j^2=-1$

Historical background

- Vishnegradsky, 1876

  (a polynomial of degree 3 with 2 parameters being coefficients)

- Neimark, 1948

  (single-input or single-output systems, transfer function is a polynomial of degree n)

$$\det(I+M(\xi(\omega))K)=0 \quad \xrightarrow{m=1} \quad \boxed{1+\sum_{i=1}^{r} M_i(\xi(\omega))K_i=0}$$

$$a_0(s)+\sum_{i=1}^{r} k_i a_i(s)=0$$

affine poly family of degree n ($A \in \mathbf{R}^{n\times n}$)

single-input single-output systems

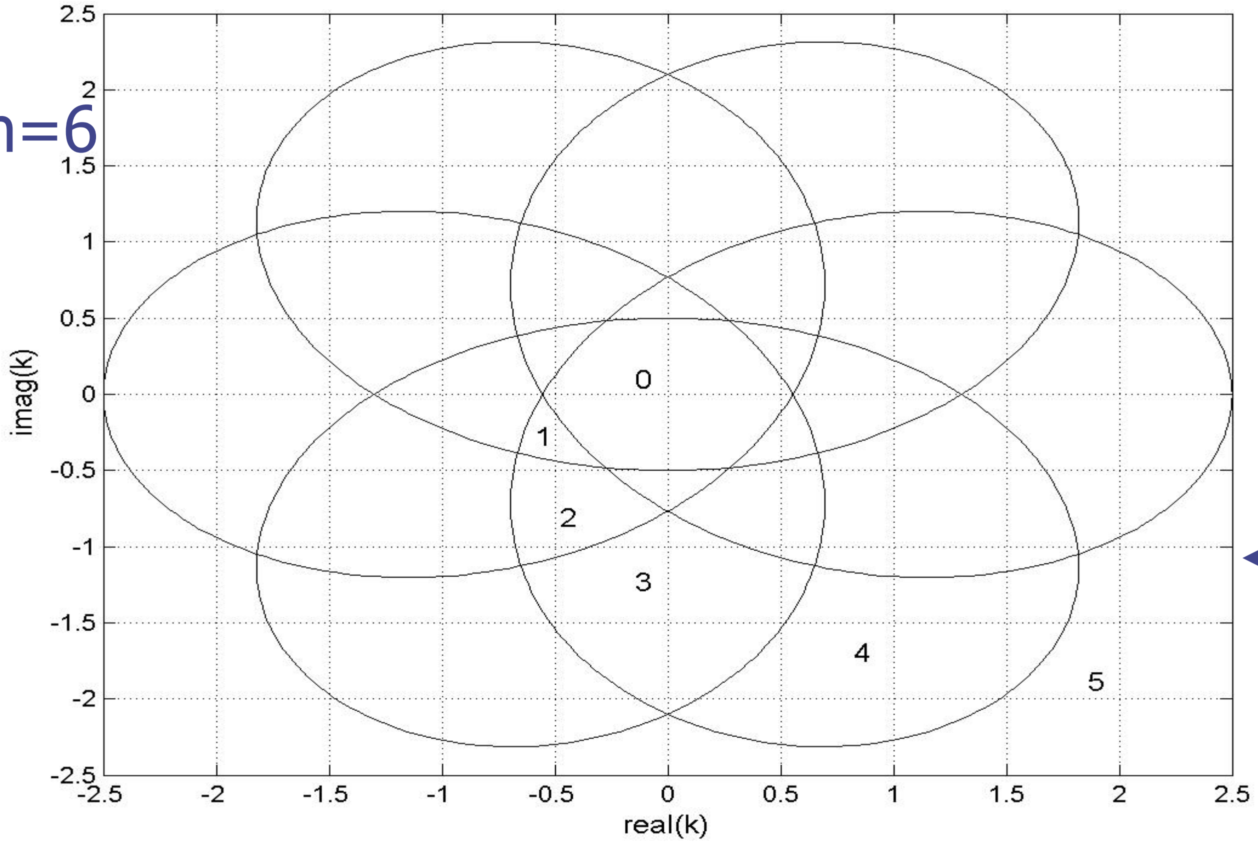
$A \in \mathbf{R}^{n \times n}$

Th: $k \in \mathbf{R}$ — **n+1** EIR, **n/2** stability intervals

$k \in \mathbf{C}$ — $\mathbf{n^2\text{-}2n\text{+}3}$ EIR

[proof is based on Bezout Theorem (algebraic geometry)]

n=6



$1+kM(\xi(\omega))=0$

$1+kz^{n-1}/(z^n+\alpha)=0$

$z=e^{j\omega}$

$\alpha<1/(n-1)$

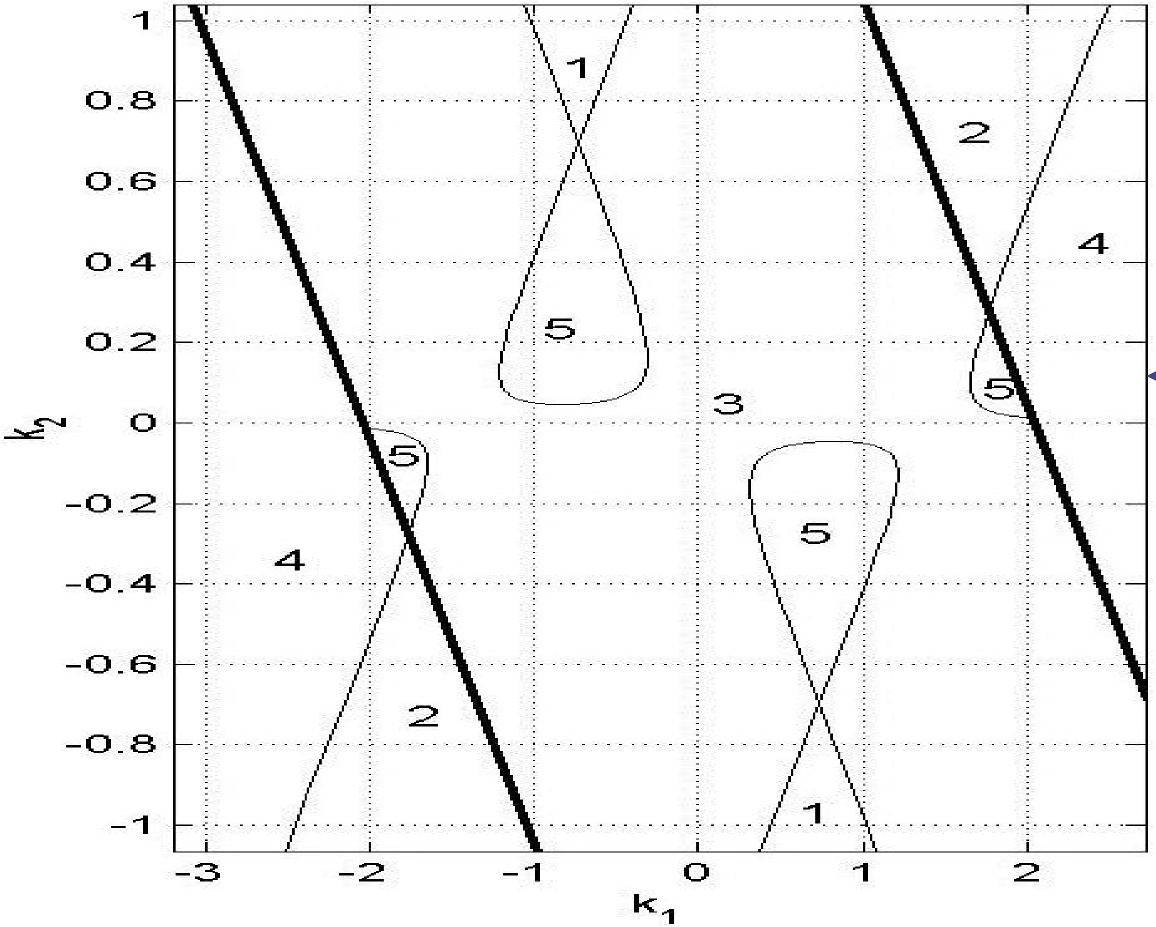
$n^2\text{-}2n\text{+}2$ EIR

single-input double-output systems

Th: $2n^2-2n+3$ EIR

$A \in \mathbf{R}^{n \times n}$

n=5 $\alpha=1.05$



$1+k_1M_1+k_2M_2=0$

$z^n + k_1 z^{n-1} + \alpha z^{n-2} + k_2 = 0$

$1<\alpha<n/(n-2)$

n-1 stability regions

Conjecture: max number of stability regions n–1

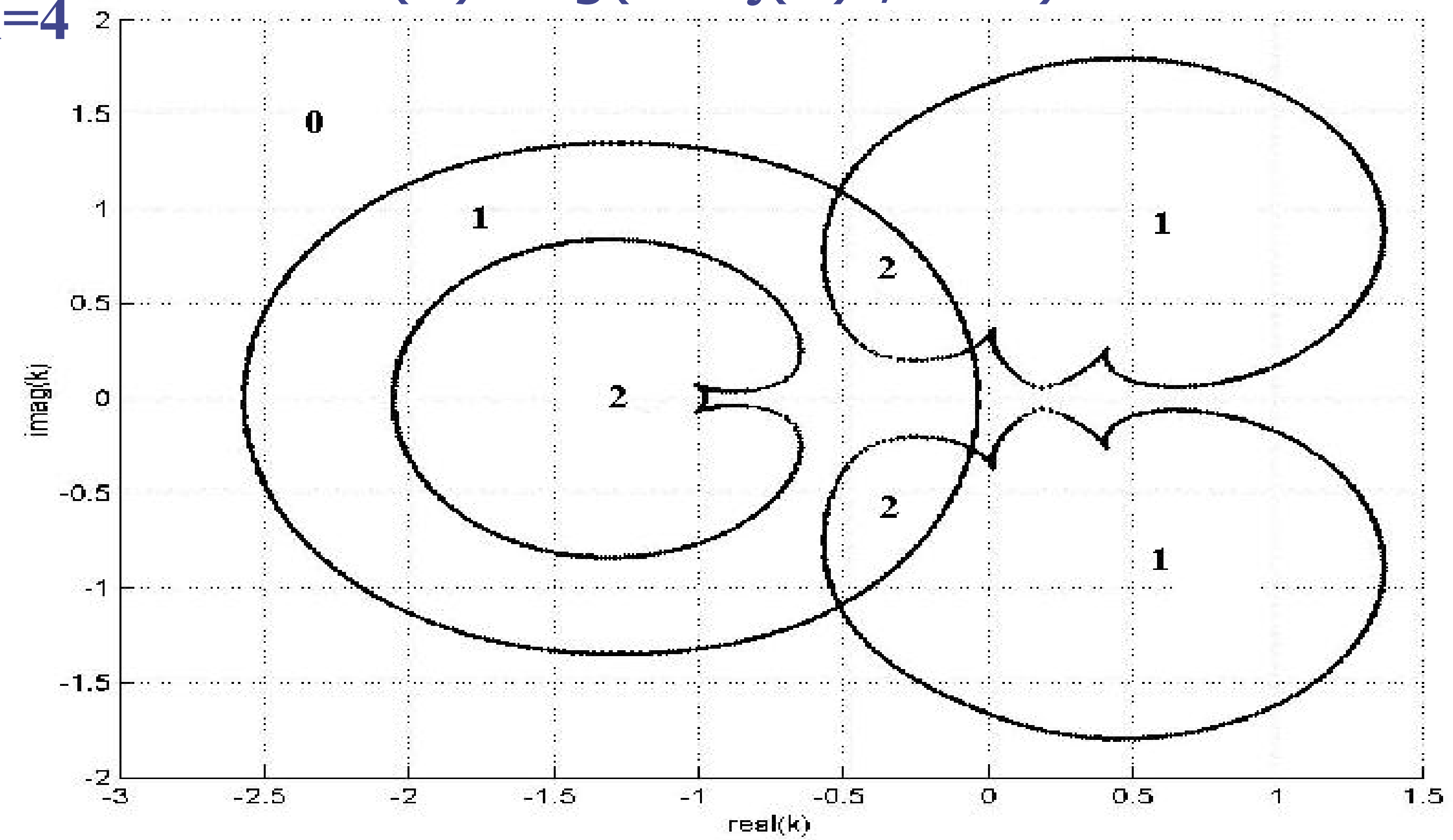
Scalar gain, $K=kI$, $k\in C$

$\det(I+M(\xi(\omega))K)=0$

$1+k/\lambda_i(\omega)=0$ $\lambda_i(\omega)$ eignvalues of $M(\xi(\omega))$

$k(\omega)=\text{eig}(A - \xi(\omega)I, - BC)$

n=4



Scalar gain, K=kI, k∈C

$k(\omega)=\text{eig}(A - \xi(\omega)I, - BC)$

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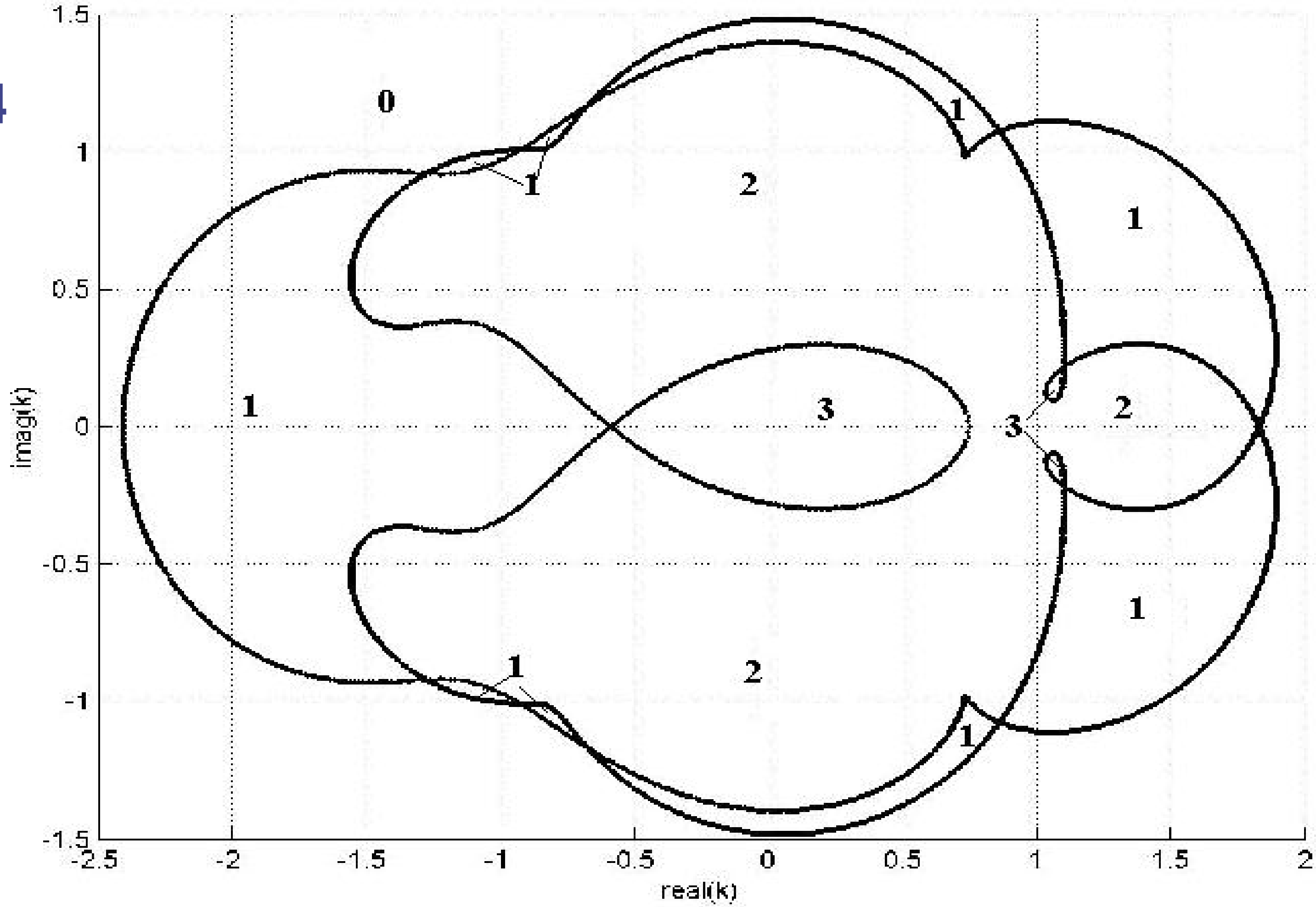


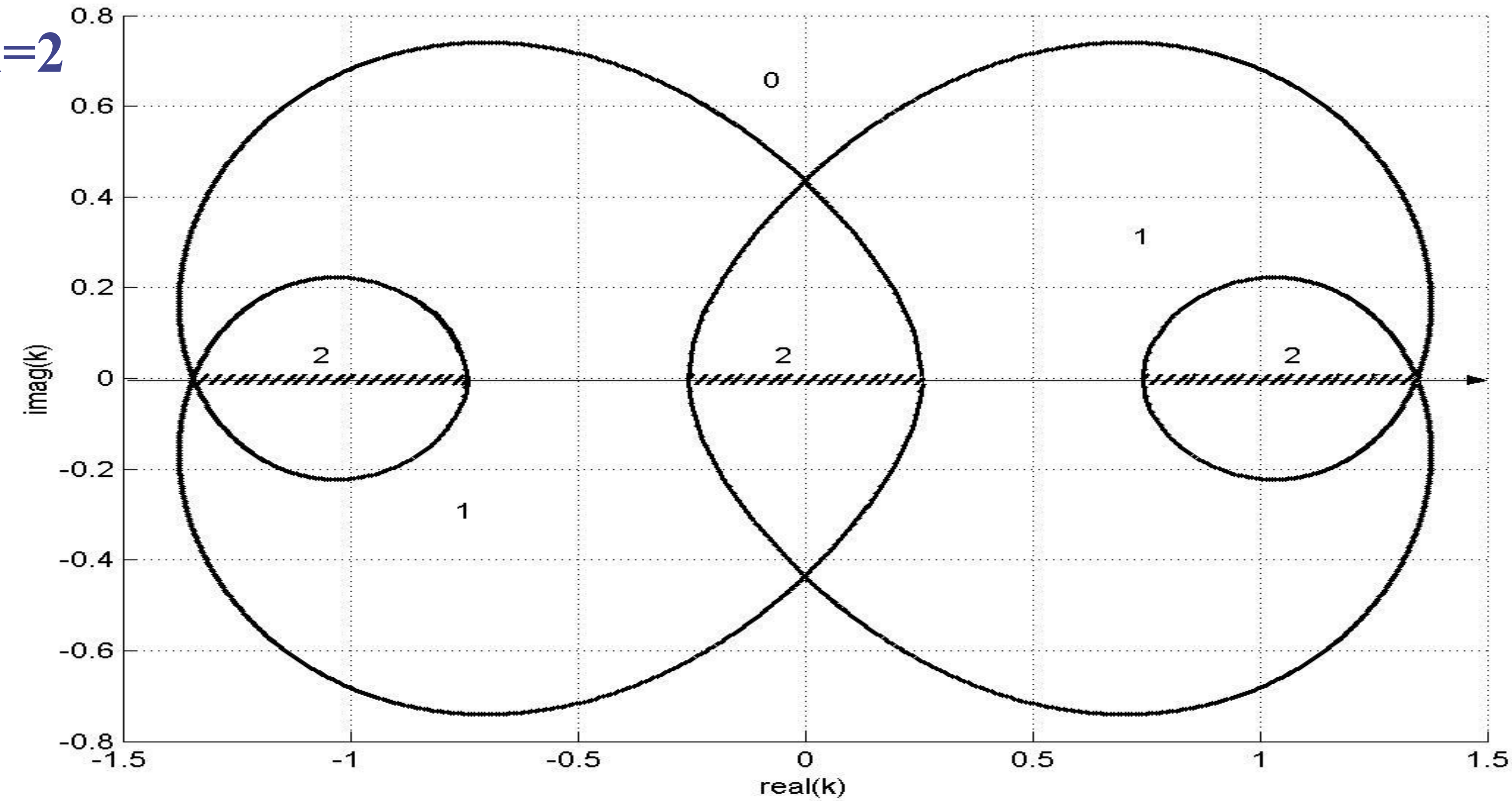
Scalar gain, $K=kI$, $k \in R$

$A \in \mathbf{R}^{n\times n}$

Th: n(n+1) EIR, **$n^2/2$** stability intervals.

compare with n+1 EIR, **n/2** stability intervals for SISO

n=2



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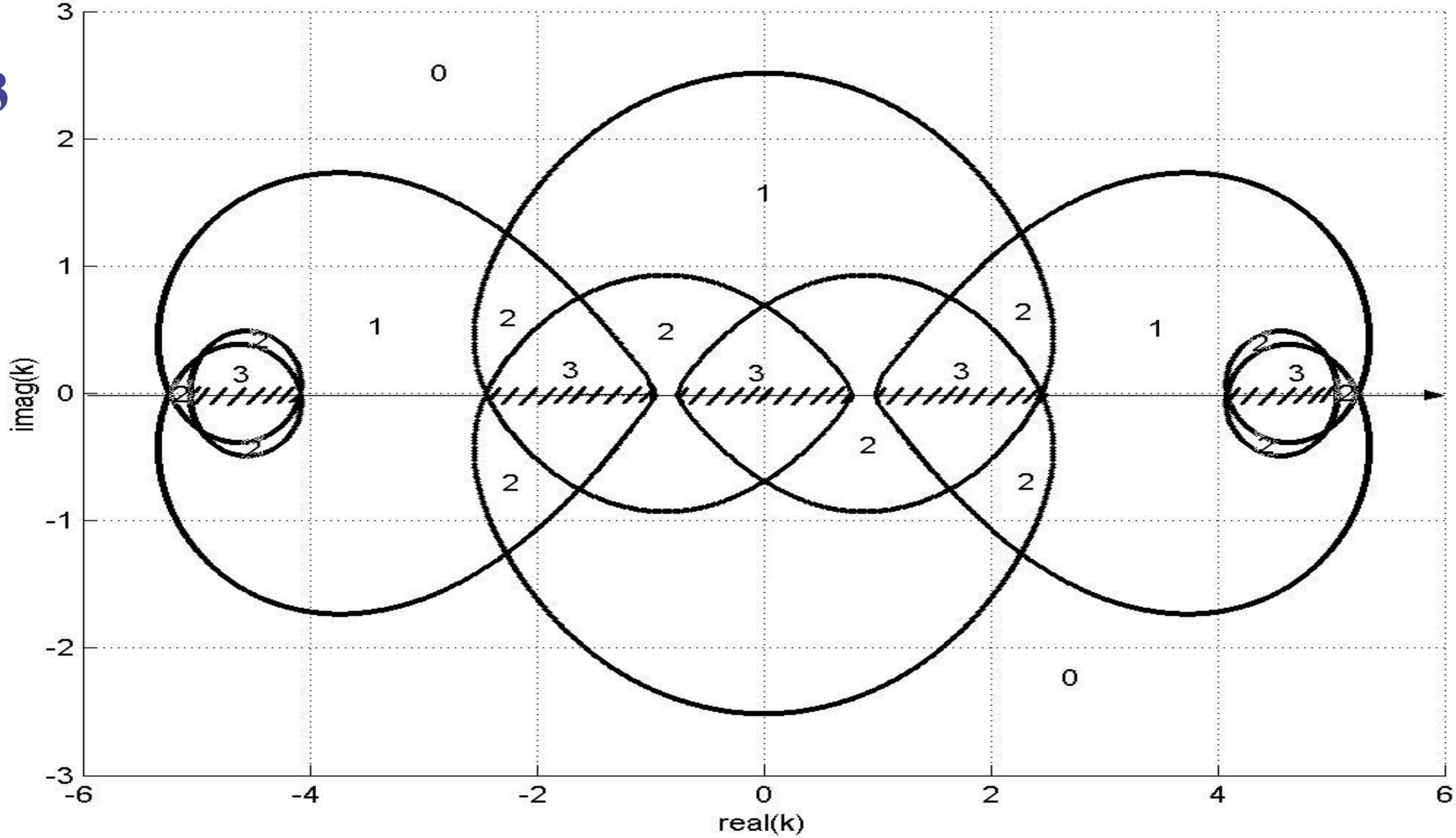
n=3



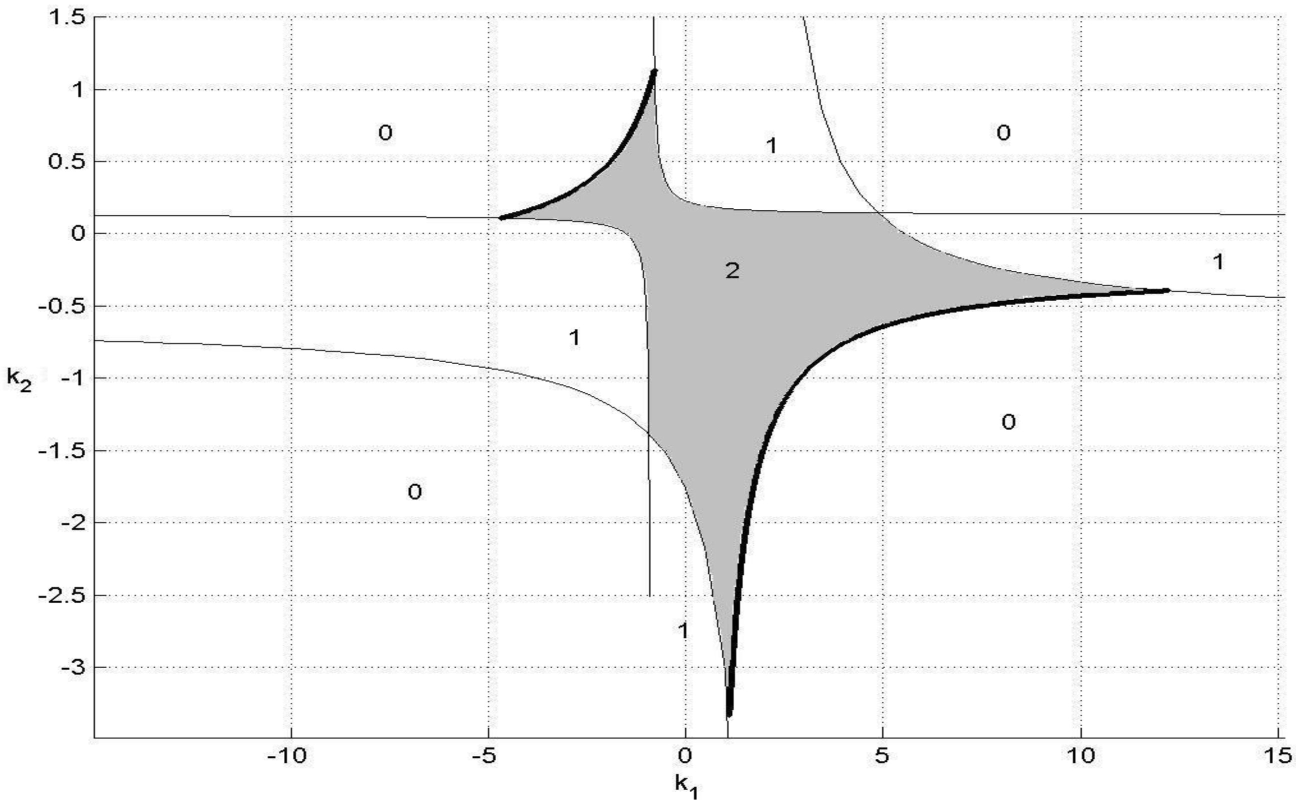
Double-input double-output systems

$\det(I+M(\xi(\omega))K)=0$

$K=\begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}$, $k_1, k_2 \in \mathbf{R}$

n=2

$\xi(\omega)=e^{j\omega}$

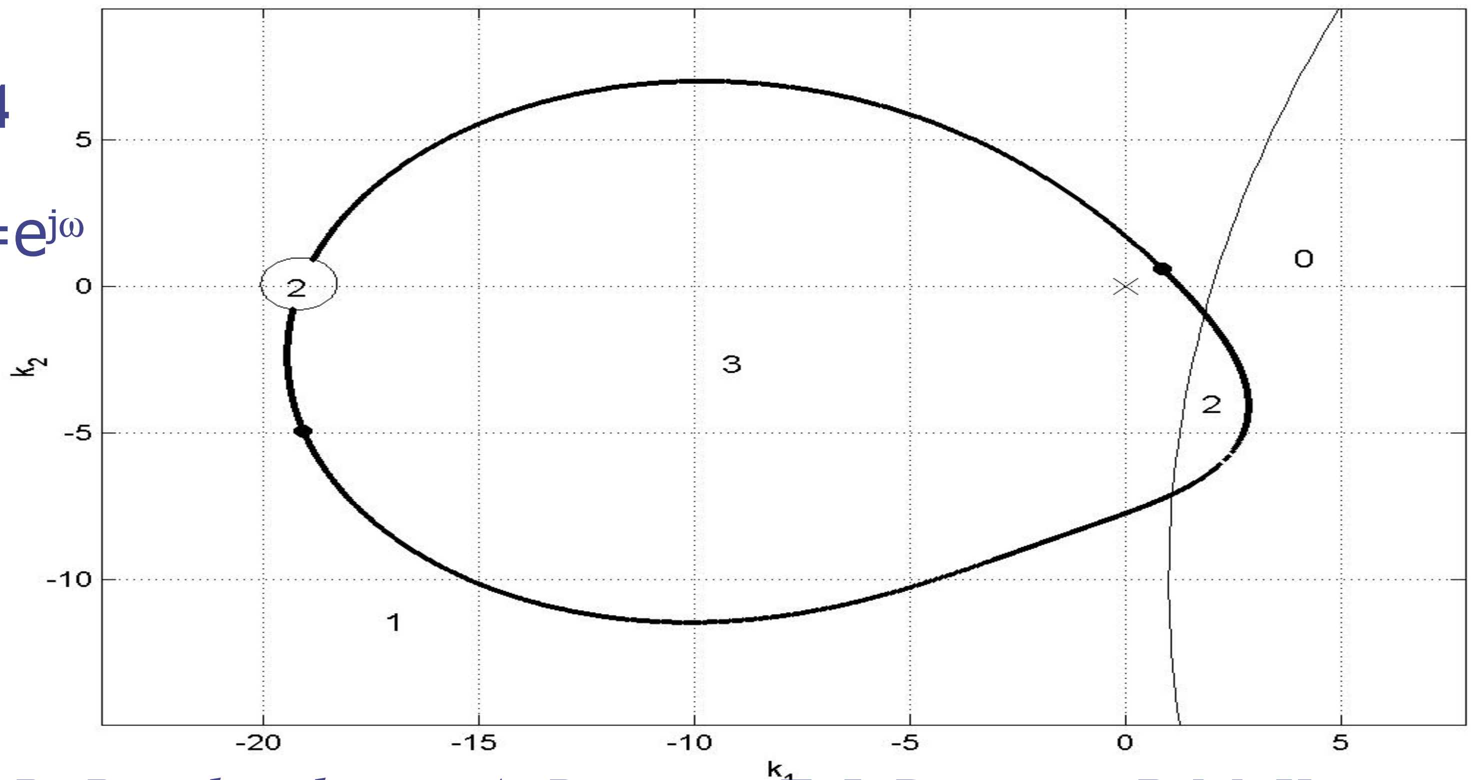


Double-input double-output systems

$\det(I+M(\xi(\omega))K)=0$ $\quad K=\begin{pmatrix} k_1+ik_2 & 0 \\ 0 & k_1-ik_2 \end{pmatrix}$ or $\begin{pmatrix} k_1 & k_2 \\ -k_2 & k_1 \end{pmatrix}$, $k_1, k_2 \in \mathbf{R}$

n=4

$\xi(\omega)=e^{j\omega}$



L. Qui, B. Bernhardsson, A. Rantzer, E.J. Devison, P.M. Young, J.C. Doyle "A formula for computation of the real stability radius" // Automatica, 1995, V. 31, No. 6, p. 879 – 890

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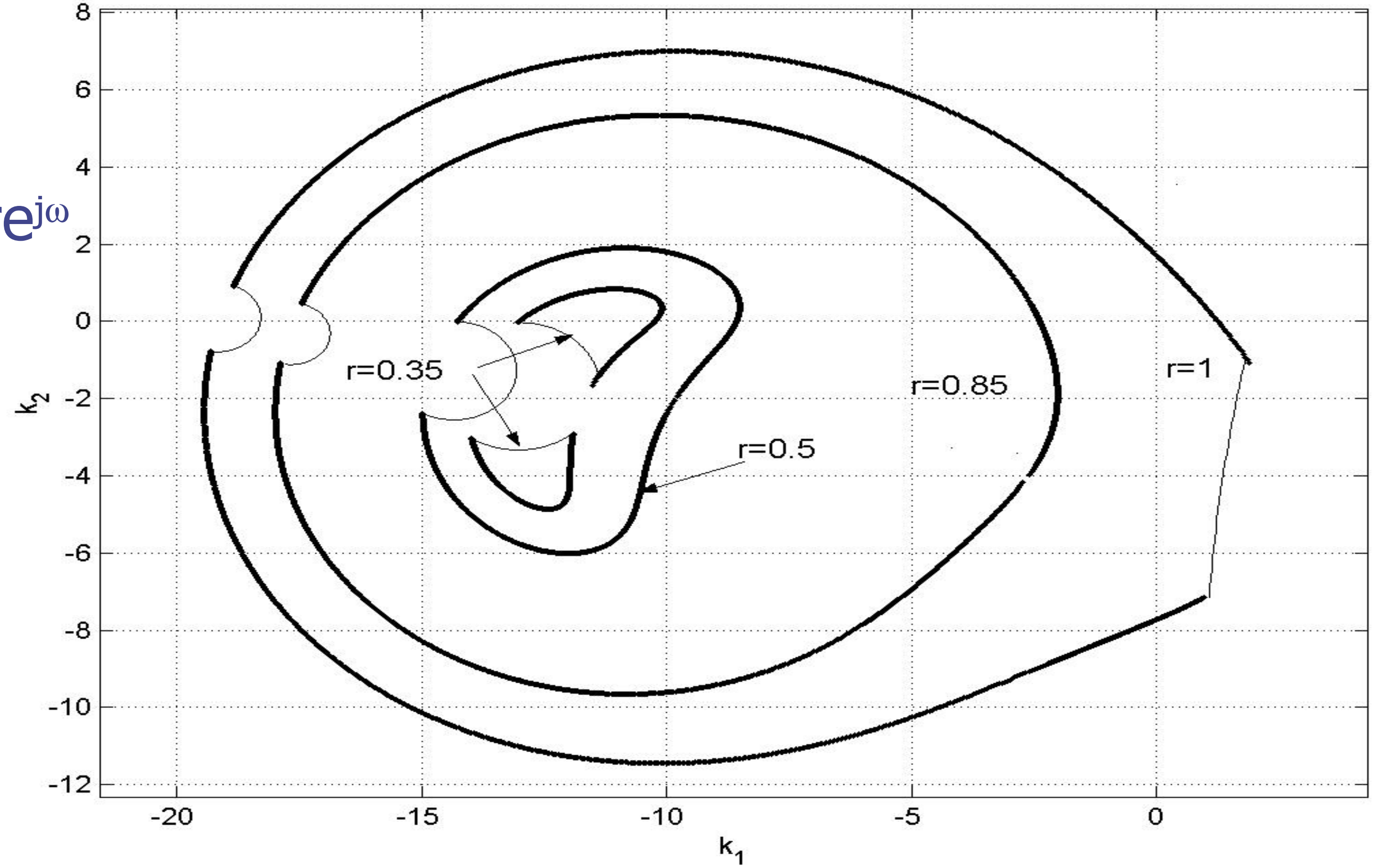

Conclusion

- A non-conservative estimation of the number of EIR for SISO system and scalar gain.
- An extension of *D*-decomposition method to double-input double-output systems with two parameters.