ON APPROACH TO THE FEEDBACK CONTROL FOR DISTRIBUTED SYSTEMS

Aida-zade K.R., Handzel A.V.

1. INTRODUCTION

In recent years with the development of technical and measurement tools of control systems more attention is paid to the problems of synthesis of control laws for objects with lumped parameters, governed by ordinary differential equations, as well as for objects with distributed parameters, described by partial differential equations [1] - [6].

In this paper we develop further the ideas of [1] and study the problem of synthesis of lumped sources control for the objects with distributed parameters on the basis of continuous observation of phase state at given points of object. In the proposed approach the phase state space (phase space) is beforehand somehow partitioned at observable points into given subsets (zones). The synthesizing control actions therewith are taken from the class of piecewise constant functions. The current values of control actions are determined by the subset of phase space that contains the aggregate of current states of object at the observable points (in these states control actions take constant values). In the paper such synthesized control actions are called zone control actions. A technique to obtain optimal values of zone control actions with the use of smooth optimization methods is given. With this aim, the formulas of objective functional gradient in the space of zone control actions are obtained.

2. THE FORMULATION OF THE PROBLEM OF LUMPED CONTROL SYNTHESIS

To illustrate the proposed approach to the study of the problem of control synthesis in distributed systems, we consider the problem of control of heat-exchanger process. Let us describe briefly the process. A liquid goes through the heat-exchanger. The temperature of the liquid at the entry point of heat-exchanger is $\psi_2(t)$. The temperature of steam in the jacket is $v(t)$. The function $v(t)$ is control parameter in the following problem:

$$u_t(x,t) = -a_1 u_x(x,t) - a_0 u(x,t) + a v(t), \quad x \in [0,l], \quad t \in [0,T] \tag{1}$$

$$u(x,0) = \psi_1(x) \in \Psi_1(x), \qquad u(0,t) = \psi_2(t) \in \Psi_2(t). \tag{2}$$

Here $u(x,t)$ is the temperature of the liquid, $\Psi_1(x), \Psi_2(t)$ are given domains of initial and boundary conditions.

Let us suppose, that there are L sensors mounted at the heat-exchanger at the points $\tilde{x}_j \in [0,l], \quad j = 1,\dots,L$. The sensors perform on-line monitoring and reading-in of information about the temperature of liquid at these points into the process control system. This information is defined by the vector:

$$u(\tilde{x}_j, t) = \tilde{u}_j(t), \; j = 1,\dots,L, \quad t \in [t_0, T].$$

The problem is to determine such control function $v(t) \in U$ with respect to current values of $\tilde{u}_j(t),\ j = 1,\dots,L$ that minimizes the functional:

$$J(v) = \alpha \int_0^T v^2(t)dt + \int_0^l [u(x,t) - u^*(x)]^2 dx\,, \tag{3}$$

Actually, we have to find the mean value of functional (3) for all possible initial and boundary conditions, i.e., the functional to be minimized is:

$$J(v) = \int_{\psi_1(x)} \int_{\psi_2(t)} \int_0^l \int_0^T \alpha_2(x)\alpha_2(t)[u(x,t;\psi_1(x),\psi_2(t),v(t)) - u^*(x)]^2 dt\,dx\,d\psi_1 d\psi_2, \tag{4}$$

here $u(x,t;\psi_1,\psi_2,v)$ is the solution of (1), (2) under specified admissible initial-boundary functions $\psi_1(x), \psi_2(t)$; the functions $\alpha_1(x), \alpha_2(t)$ are weight functions. In the case of uniform admissible initial-boundary conditions weight functions are determined as follows:

$$\alpha_1(x) = 1/mes\Psi_1(x), \qquad \alpha_2(t) = 1/mes\Psi_2(t).$$

The condition (2) has the following meaning. During operation of heat-exchanger, current control must not "remember" exact initial-boundary conditions, conversely, the control must be tuned at their "mean" values and must depend only on the states at the observable points:

$$v(t) = v(t;\tilde{u}_1(t),\dots,\tilde{u}_L(t)). \tag{5}$$

We shall assume that the functions involved in the formulations satisfy all the necessary conditions of existence and uniqueness of the solution of the considered problem.

This formulation resembles the classical one, but here we consider a more general case when $v(t)$ can not change its value at any time instance, instead, $v(t)$ remains constant till the values of all observable states remain in some definite zone. We suppose here that the set of all admissible values of phase variable is partitioned into zones, i.e.:

$$\underline{u} \le u(x,t;\psi_1,\psi_2,v) \le \bar{u}, \quad \forall\,(\psi_1,\psi_2,v(t))\,, \tag{6}$$

here $\underline{u}, \bar{u}$ are usually known values, determined by operational consideration. Then it holds for observable states as well:

$$\underline{u} \le \tilde{u}_j(t) \in \bar{u}, \quad j = 1, L. \tag{7}$$

Let us partition the space of observable states into intervals $[\dot{u}_{i-1}, \dot{u}_i)$ by the values: $\underline{u} = \dot{u}_0 < \dot{u}_1 < \dots < \dot{u}_m = \bar{u}$. These values determine the zones of state values at the observable points. So we have:

$$\dot{u}_{i_1-1} \le \tilde{u}_1(t) < \dot{u}_{i_1},\ \dot{u}_{i_2-1} \le \tilde{u}_2(t) < \dot{u}_{i_2}, \dots, \dot{u}_{i_L-1} \le \tilde{u}_L(t) < \dot{u}_{i_L} \tag{8}$$

The sets (8) constitute L - dimensional parallelepipeds in L - dimensional phase state space $\tilde{u}_j(t)$ $j = 1,\dots,L$. The total number of these parallelepipeds is m^L.

It is clear that feedback with the object (i.e. sensing) is required to determine whether the state at the observable points falls into the $(i_1,\dots,i_L)$-th zone. The aggregate of all zones covers all possible values of observable states. It is also clear that control functions (5) change its values only at those time instants when the set of states at the observable points passes from one phase parallelepiped (8) to another one.

The problem consists in determination of constant value of $v(t)$ for each zone while the states of all observable points remain in this zone, i.e. the following holds:

$$v(t) = v(\tilde{u}_1(t),\dots,\tilde{u}_L(t)) = v_{i_1,\dots,i_L} = const, \quad t \in [0,T] \tag{9}$$

while (8) holds.

The number of different values attained by steam temperature equals to the number of phase parallelepipeds defined by inequalities (8), i.e., m^L. The total number of parameters to be optimized is also m^L. They determine the value of steam temperature for all possible values of liquid temperature at the observable points.

So, the considered problem of heat-exchanger process control with the use of feedback at the class of piecewise constant functions consists in optimization of m^L - dimensional vector:

$$v = (v_{\underbrace{1\dots1}_{L}},\dots,v_{\underbrace{i_1\dots i_L}_{L}}\dots,v_{\underbrace{m\dots m}_{L}}), \tag{10}$$

which directly determines the course of heat-exchanger process and, consequently, the value of objective functional (11).

3. SOLVING PROBLEM OF ZONE CONTROL SYNTHESIS

Original problem of optimal control of distributed object without condition (9) on the class of control actions was studied in many papers [7]. The assumed in this work condition of piecewise constancy of control (9) leads to parametric problem of optimal control of distributed system, in which finite-dimensional vector of parameters (10) is to be optimized. On the other hand, the obtained problem may be considered as finite-dimensional optimization problem. Its objective function depends on optimized vector v and is determined by solving boundary value problem (1), (2) and by computation of functional (3) or (4).

Taking into consideration the aforesaid, for numerical solving zone control synthesis problem, we can use the methods of finite-dimensional smooth optimization, in particular, iterative method of gradient projection type:

$$v^{(q+1)}(\alpha_q) = P_U(v^{(q)} - \alpha_q gradJ(v^q)), \quad q = 0, 1, \dots, \tag{11}$$

$$\alpha_q = \arg\min_{\alpha>0} J(v^{(q+1)}(\alpha)).$$

Here: v^0 is some given value of initial guess vector of zone control actions v , $P_U(z)$ is the operator of projection of element z on admissible set U . Let us present the obtained formulas of functional gradient in the space of optimized parameters.

One of important elements in calculating gradient is the time interval when phase state $\tilde{u}(t)$ belongs to the one or another phase parallelepiped. Let us denote by $\Pi_{i_1,...,i_L}(\psi_1,\psi_2,v) \in [0,T]$ the time period during which (8) holds, $i_1 = \overline{1,m}, ..., i_L = \overline{1,m},$ i.e. $i_s = 1,...,m,\quad s = 1,...,L$ (the dependence of $\Pi_{i_1,...,i_L}$ on ψ_1,ψ_2,v is evident).

In classical formulation the adjoint system with respect to $\mathrm{P}(x,t) = \mathrm{P}(x,t;\psi_1,\psi_2,v)$ under fixed initial-boundary conditions and fixed control is as follows:

$$\mathrm{P}_t^{'} = -a_1 P_x^{'} + a_0 P \tag{12}$$

$$\mathrm{P}(x,T) = 2(u(x,T) - u^*(x)) \tag{13}$$

$$\mathrm{P}(l,t) = 0 \tag{14}$$

The following theorem holds.

Theorem 1. The components of functional gradient in the problem (1), (2), (3) in the space of piecewise constant control actions (9) for arbitrary control $v \in U$ under appropriate normalization are defined by the formula:

$$\frac{dJ}{dv_{i_1,...,i_L}} = a_0 \int\limits_{\psi_1} \int\limits_{\psi_2} \int\limits_0^l \int\limits_{\Pi_{i_1,...,i_L}} P(x,t)dtdxd\psi_2 d\psi_1 \tag{15}$$

here $P(x,t)$ is the solution of adjoint problem (12) – (14), that corresponds to current zone control.

The formula (15) allows us to compute the functional gradient at current value of control actions and to use it further in iterative procedures of smooth optimization, for example, (11).

We must note, that actual dimensions of optimized parameters vector are rather large. So, a lot of calculations and computer time may be needed to solve control synthesis problem, presented in the paper. Taking into consideration, on the one hand, the importance and complexity of control synthesis problem, and, on the other hand, the absence of demand of these problems' real-time solving, we think that the proposed approach may be applied when designing a number of automatic and automated systems of control of technical objects and processes.

Let us emphasize important advantage of the proposed in the paper synthesized zone control as compared to synthesized classical control of $v(u(x,t))$ type. It is caused by technical complexity of on-line acquirement of information about current states of objects at the observable points and construction of control actions on the basis of this information. Whereas, the construction of zone control is performed during the time, when the states of object lie in the definite zones of phase space. This argues for robust features of zone control, proposed in the paper [4].

Also it is of practical interest to apply the proposed approach to constructing control actions from the results of observation of object current state values only at certain points of object. In many cases, this corresponds to actual capacities of measurement systems at objects.

And, finally, it is clear, that one can easy extend the study of illustrative problem of heat-exchanger control to a number of other problems of control of distributed objects, governed by functional equations of another type.

4. NUMERICAL EXPERIMENTS.

The proposed method of the considered problem solving was applied to some test problems. When executing numerical experiments, poor convergence to the solution of the problem (1-3) has been revealed. Probably, this fact is linked with the possibility of abrupt variation of $v(t)$ when trajectory transits from one phase parallelepiped to another if the values of piece-wise constant control actions assigned to these parallelepipeds are essentially different. To overcome this the calculations were performed in two stages. At the first stage we used a control function the value of which for a given fixed trajectory is equal to a weighted linear combination of all values of control actions for all phase parallelepipeds. The weighted coefficients for the control $v_{i_1 \cdots i_L}$ were inversely proportional to the distance from the current trajectory to the $i_1 \ldots i_L$ -th phase parallelepipeds, i.e.:

$$v_{sm}(t) = \sum_{i_1=1}^{m} \ldots \sum_{i_L=1}^{m} \lambda_{i_1 \ldots i_L} v_{i_1 \ldots i_L} \ , \tag{16}$$

$\lambda_{i_1 \ldots i_L} = e^{-\frac{\rho^2_{i_1 \ldots i_L}}{2\sigma^2}}$, here $\rho_{i_1 \ldots i_L}$ is the distance from trajectory *u(x,t)* to the $i_1, \ldots, i_L$ –th phase parallelepiped. Control in the form (16) we will call "smoothed control", since its value changes smoothly when trajectory moves from one phase parallelepiped to another. The less is σ , the less is the influence of control assigned to the phase parallelepipeds that do not contain trajectory at the current time instance in total control (16). That is why in the course of optimization search we can decrease σ , thus approaching to the problem in its source formulation. At the end of optimization we can use optimal functions $v_{sm}(t)$ determined at the previous steps as initial points and try to solve the problem with piece-wise constant $v(t)$.

Numerical experiments were carried out for three problems with different terminal function *u*(x)*. Common data for all three problems are:

Equation coefficients: $a_1=1$, $a_0=0.5$, $a=1.5$; length and time limits: $L=1$, $T=1$; regularization coefficient: $\alpha = 4.0 * 10^{-4}$; the points of observation: $x_1=0.1$, $x_2=0.4$, $x_3=0.6$, $x_4=0.9$; intervals of space partition by u: $\check{u}_0 = -99999$, $\check{u}_1 = 1.7$, $\check{u}_2 = 2.2$, $\check{u}_3 = 99999$; implicit method was used for solving boundary value problem (1),(2). The following problems were solved.

Problem I.

Boundary and initial conditions: $\Psi_1(x)=1$, $\Psi_2(t)=1$, the number of discretization points along *x* and *t* is 11 (Nx=11, Nt=11).

The terminal function *u*(x)* is the solution *u (x,t)* of (1)-(2) when *t=T* under some given $v^{optimal}(t)$. Then we "forget" $v^{optimal}(t)$, and start solving the problem as if we do not know the solution. If we will be able to find $v^{optimal}(t)$, then it can testify for correct algorithm of the problem solving.

The results of numerical experiments are shown in the figures 1,2. The figures show the values of liquid temperature at the terminal time instance *T* and the values of function $v(t)$ when *t* changes from 0 till 1. Notation "without smoothing, without regularization" denotes the solution obtained without application of (16) and when $\alpha=0$.

Problem II.

The case of linear *u*(x)*: *u*(x)=1+2.5 x*. Here again $\Psi_1(x)=1$, $\Psi_2(t)=1$.

The results of the problem solving are given at the figures 3,4. In numerical experiments for the problem II we studied the influence of increase of the number of discretization along *x, t* on the control behavior and on trajectory approaching to the given terminal trajectory *u*(x)*.

Problem III.

The case of constant *u*(x)*: *u*(x)= 2.* Here $\Psi_1(x)=1$, $\Psi_2(t)=1+t$, the number of discretization points along *x* and *t* is 21 (Nx=21, Nt=21).

The results of the problem solving are presented at the figures 5,6. Notation "smoothing and regularization" means application of (16) and that the regularization coefficient α is not equal zero. In numerical experiments for this problem one of the best solutions is the solution obtained from initial point that was optimal for the problem with decreased number of discretization points along *x* and *t* ("The best Nu obtained from the best Nu when nx=nt=11").

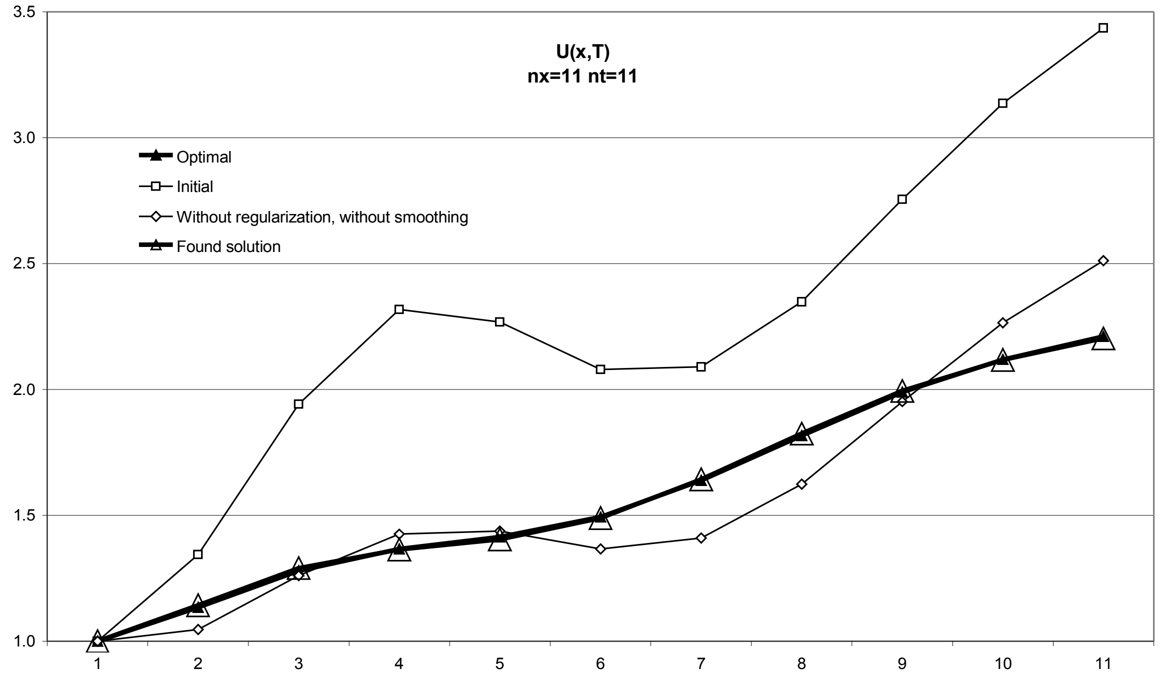

Figure1.

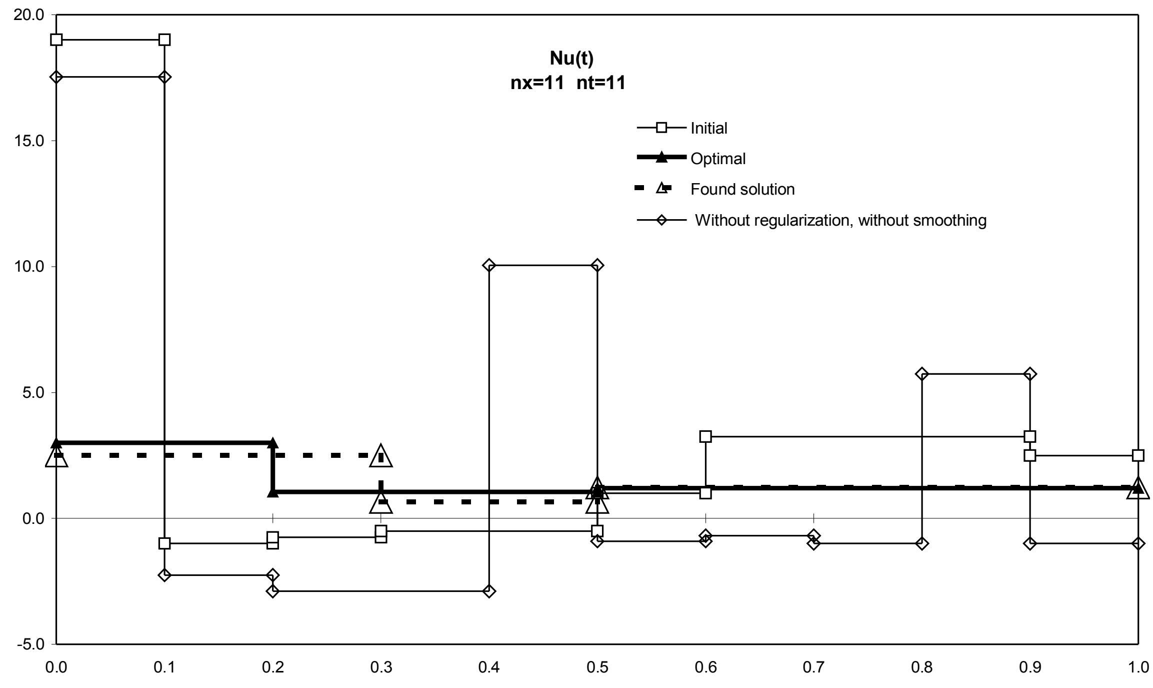

Figure 2.

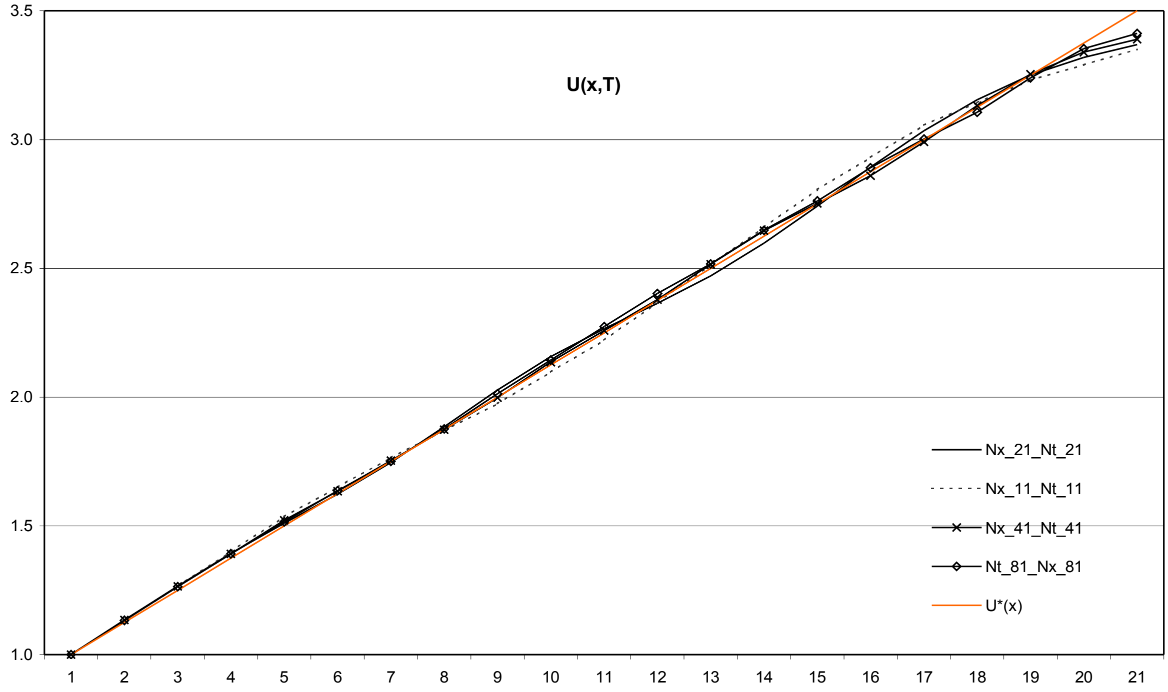

Figure 3

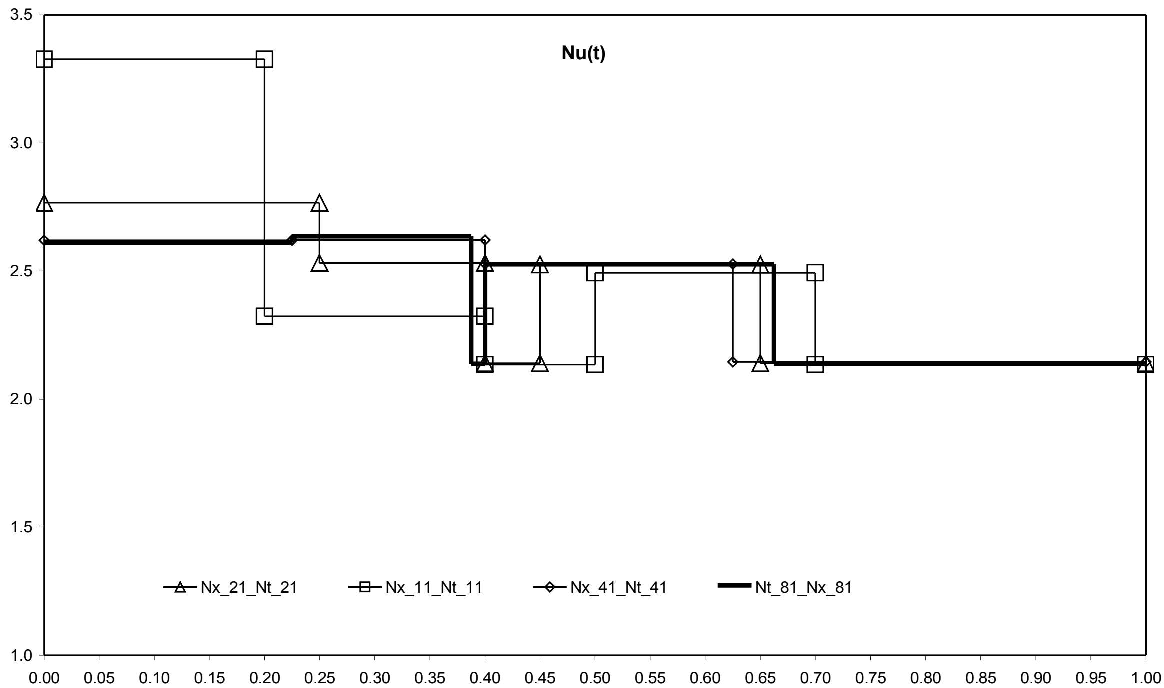

Figure 4

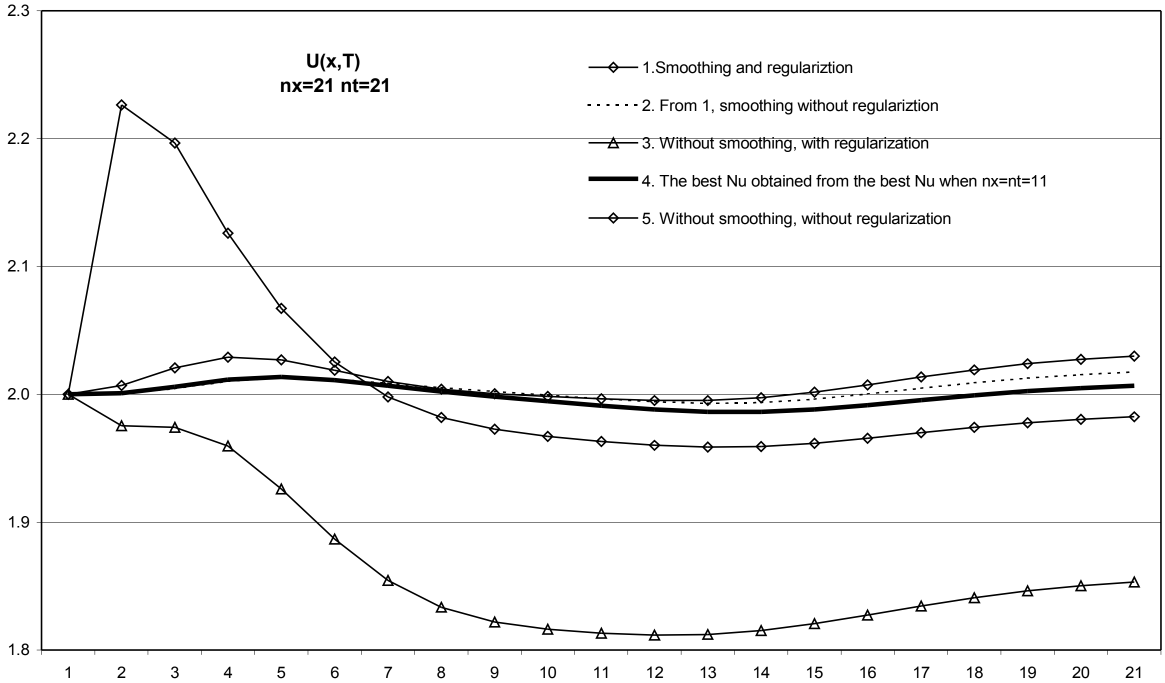

Figure 5

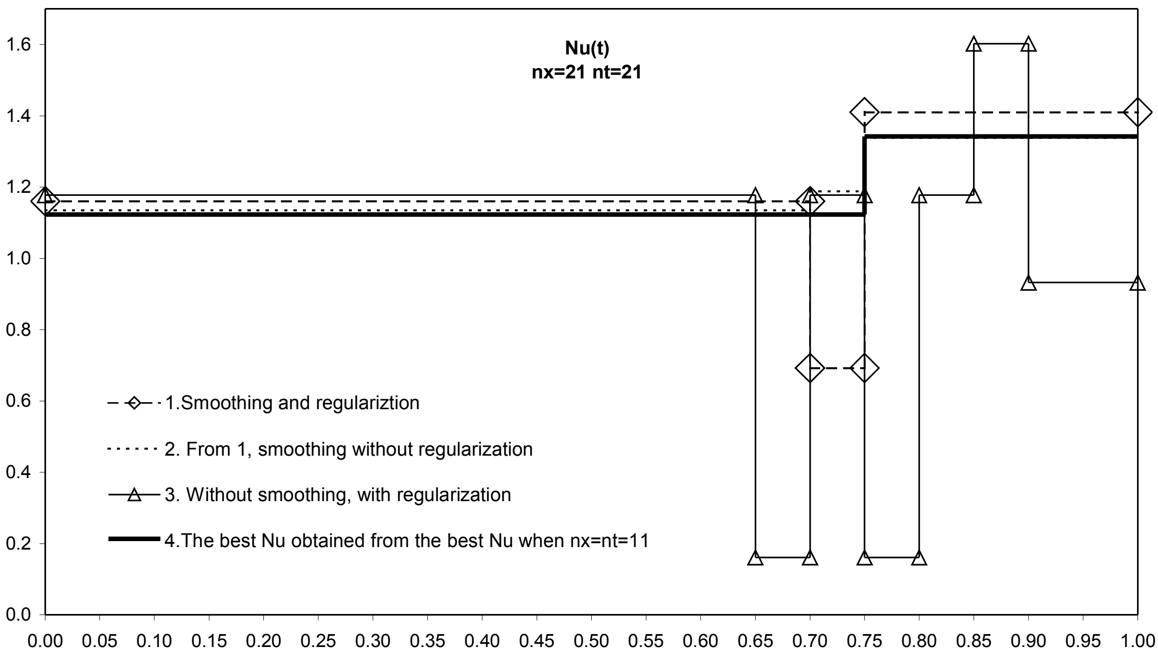

Figure 6

REFERENCES

[1] Aida-zade K.R. The problem of control in the mean with respect to regional control actions. Transactions of the National Academy of Sciences of Azerbaijan, №4, 2003.

[2] Ivanova A.P. Feedback control for stochastic heat equation. Journal of Computer and System Sciences International. Vol. 42, No. 5, 2003 pp.683-691.

[3] Systems and control encyclopedia . Ed. M.G.Singh. v.1-8. Pergamon Press,1987.

[4] Polyak B.T., Scherbakov P.S. Robust stability and control. Moscow, «Nauka», 2002.

[5] The control handbook. Ed. W.S. Levine. CDC. Press. IEEE Press,1996.

[6] W. Harmon Ray. Advanced Process Control. McGraw-Hill, 1981.

[7] Lions I.L. Control optimal de systemes gouvernes par des equations aux derivees partielles . Dunod Gauthier – Villars, Paris, 1968.