STACKLEBERG-NASH EQUILIBRIUM IN CONFLICTS WITH A LEADER: NUMERICAL PROCEDURE

A. S. Poznyak
apoznyak@ctrl.cinvestav.mx
Dept. of Automatic Control, CINVESTAV, IPN, México

The conflict situation among multi-participants with a leader who has a preference to be the first in the turn of an action selection, is tackled. When the strategy of a leader (player 1) is selected, the rest of participants are playing a "standard" noncooperative finite game (may be, with constraints) trying to find a *Nash equilibrium.* To guarantee the uniqueness of this equilibrium, the so-called δ-*regularized individual pay-off function* is introduced. Then the generalized version of the *Mangasarian-Stone theorem* is applied permitting to reformulate this non-cooperative game as a poly-linear programming problem. The *first main result* of this work consists of the theorem which shows that the last nonlinear programming problem may be represented as a *linear programming problem* (LPP) formulated in term of counter-coalition strategies. Finally, when the Nash-equilibrium strategies (as a functions of the strategy selected by a leader) are found, the leader optimizes his own pay-off solving the following optimization problem:

$$J_1\big(p^{(1)}, p^{(2)*}(p^{(1)}), ..., p^{(N)*}(p^{(1)})\big) := \mathring{J}_1(p^{(1)}) \longrightarrow \min_{p^{(1)} \in Q^{(1)}_{admis}}$$

where $p^{(i)*}(p^{(1)})$ are the Nash equilibrium strategies for the rest of participants. The application of something like-gradient technique to solve this optimization problem seems to be not so simple task since the sensitivity matrices

$$\frac{\partial}{\partial p^{(1)}} p^{(i)*}(p^{(1)}), \qquad i = 2, \ldots, N,$$

are practically unavailable. Additionally, the functions $p^{(i)*}(p^{(1)})$ are not always differentiable. Instead, the randomized procedure

$$p^{(1)}_{n+1} = \pi_{S^1}\Big\{p^{(1)}_n - \frac{\gamma_n}{\alpha_n}\xi_n \mathring{J}_1(p^{(1)}_n + \alpha_n \xi_n)\Big\}$$

is applied, where ξ_n is a random vector uniformly distributed over unitary sphere. In fact it is a sort of a randomized quasigradient procedure. This is *the second main contribution* of the paper. Numerical examples show the workability of the approach.