ABOUT STATIONARITY AND REGULARITY OF SET SYSTEMS

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Starting with the pioneering work by Dubovitskii and Milyutin it is quite natural when dealing with optimality conditions to reformulate optimality in the original optimization problem as a (some kind of) extremal behaviour of a certain system of sets. Considering set systems is a rather general scheme of investigating optimization problems. Any set of "extremality" conditions leads to some optimality conditions for the original problem.

When the sets are convex (or admit some convex approximations) extremality conditions are given by the *separation theorem*. In the general case a nonconvex version of the separation theorem was proved in [1]. By now it is generally referred to as the *Extremal principle* and has numerous applications to optimization, calculus and economics.

Any necessary optimality conditions characterize in the nonconvex case not only optimal solutions but some broader set of *stationary* points which can also be of interest. Once the primal space description of stationary points is formulated, the (dual) necessary conditions become also sufficient. For instance, considering *weak stationarity* leads to the *Extended extremal principle* [2, 3].

When a stationarity condition is not true one can speak about (some kind of) *regularity* of the set system. A strong regularity property which corresponds to the absence of weak stationarity appears to be equivalent to the *metric regularity* of some multifunction.

References

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