THE UNIFORM DISTRIBUTION: OPTIMALITY/SUBOPTIMALITY

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When analyzing technical systems the engineers are often needed to take into account that it is known about system parameters only that they belong to given intervals of uncertainty. In this case often there is used a probabilistic approach which is based on interpretation of the uncertain parameters as random variables and on further application of the Monte Carlo method to estimate probabilistic characteristics of the system performance. The probability distribution of each such a random variable is often believed to be uniform within the corresponding uncertainty interval. A satisfactory mathematical justification of such an approach was obtained on the eve of the 21th century and it is known now as the Barmish–Lagoa Uniformity Principle.

Our interest to that principle was caused in 1996 by B. T. Polyak who introduced us to the original formulation and stated the problem of obtaining an alternative proof and its extensions.

The essence of the uniformity principle can be illustrated by the following probabilistic optimization model. There is an n-dimensional unique cube $\{x \in R^n : \ \|x_i\| \leq 1, \ i = 1, \dots, n\}$. The cube contains a convex target set which is symmetric with respect to the origin. There is the probability distribution on the cube which corresponds to an n-dimensional random vector with the independent components such that each of them has a probability density function belonging to the class of the quasi-concave even functions supported on $[-1, 1]$. Let us consider the minimization problem for the probability measure of the target set with respect to the probability density functions within that class. Then the minimum is attained at the uniform distribution over the cube.

The report is based on source publications [1–3] and contains the surway concerning both the justification of the uniformity principle and its extensions connected to weakening the independence condition, the assumption about the geometry of the target set and the requirements to the feasible distributions.

There is a natural question about the sensitivity of the uniformity principle with respect to violations of original assumptions. We overview the results on the sensitivity analysis and suggest analytic estimates of sensitivity which show that the uniform distribution is suboptimal in the above sense under the small violations.

References

[1] Kibzun, A. I., On the worst-case distribution in stochastic optimization problems with the probability function, *Avtomatika i Telemekhanika*, 1998, 11, 104–116 (in Russian).

[2] Kan, Yu. S., On the justification of the uniformity principle for the optimization problem with probabilistic objective, *Avtomatika i telemekhanika*, 2000, 1, 54–70 (in Russian).

[3] Kan, Yu. S. and A. I. Kibzun, Sensitivity Analysis of Worst-Case Distribution for Probability Optimization Problems, in: *Probabilistic Constrained Optimization: Theory and Applications* (S. P. Uryasev, ed.), Kluwer Academic Publishers, 2000, pp. 31–46.