Suboptimal robust control of unknown first order plant with delay

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General problem :

Prior information in robust control theories includes a **nominal model** of controlled plant and **weights** of uncertainties and exogenous disturbances.

How to apply results obtained in robust control theories under incomplete prior information?

In the present paper this problem is discussed for the first order plant in the ℓ_1 setup.

Plant :

$$y(t) = ay(t-1) + bu(t-d) + v(t) \quad \forall t \in \mathbb{N}$$

$y(t) \in \mathbb{R}$ – output, $u(t) \in \mathbb{R}$ – control

total disturbance:

$$v := \delta_w w + \delta_y \Delta_1 y + \delta_u \Delta_2 u \ ,$$

w – exogenous disturbance :

$$\|w\|_{\ell_\infty} = \sup_t |w(t)| \leq 1$$

Δ_1, Δ_2 – perturbations :

$$|(\Delta_1 y)(t)| \leq \sup_{t-\mu \leq s < t} |y(s)|$$

$$|(\Delta_2 u)(t)| \leq \sup_{t-\mu \leq s < t} |u(s)|$$

$\delta_w, \delta_y, \delta_u$ – upper bounds

Prior information

unknown a and b :

$$\xi := (a,b)^T \in \Xi = [\,\underline{a}\,,\bar{a}\,] \times [\,\underline{b}\,,\bar{b}\,]$$

unknown upper bounds :

$$\delta := (\delta_w, \delta_y, \delta_u)^T \geq 0$$

Control citerion

$$u = K(y) \text{ -- controller}$$

$$r(t) \text{ -- bounded reference signal}$$

$$\|y - r\|_{ss} := \limsup_{t \to +\infty} |\, y(t) - r(t)\,|$$

$$J(K, \xi, \delta, \mu) := \sup_{\Delta_1, \Delta_2} \ \sup_{w} \ \|y - r\|_{ss}$$

Control of known system

Theorem 1. 1) Given $|b| \geq \delta_u$, the controller $K_{opt}(\xi)$

$$\alpha_0(q^{-1})u = \beta_0(q^{-1})y + \frac{1}{b}q^d r\,,$$

$$\alpha_0(\lambda) = 1 + a\lambda + \ldots + a^{d-1}\lambda^{d-1}\,,\ \beta_0(\lambda) = -\frac{a^d}{b}\,,$$

minimizes the control criterion

$$J_F := \sup_{\Delta \in D_F} \sup_{\|w\| \leq 1} \limsup_{t \to \infty} |\, y(t) - r(t)\,|$$

(D_F - fading (finite) memory perturbations)

$$\min_K J_F = \left(\delta_w + \delta_y\|r\|_{ss} + \delta_u \frac{\|(1 - aq^{-1})r\|_{ss}}{b}\right) \times$$

$$\frac{1 + a + \ldots + a^{d-1}}{1 - \delta_y(1 + a + \ldots + a^{d-1}) - \delta_u|a|^d/|b|} =: J_{opt}(\xi, \delta)$$

2) $J(K, \xi, \delta, \mu) \nearrow J_{opt}(\xi, \delta)$ as $\mu \to +\infty$ if r gets into neighborhoods of its upper limit uniformly often.

Problem statement

data :

$$y_0^t = (y(0), \cdots, y(t)),$$

$$u_0^{t-1} = (u(0), \cdots, u(t-1))$$

To find a controller K of the form

$$u(t) = K(y_0^t, u_0^{t-1}, r) \quad \forall t \in \mathbb{N}$$

for the system with unknown parameters ξ and δ such that

$$\limsup_{t \to +\infty} |\, y(t) - r(t) \,| \leq J_{opt}(\xi, \delta),$$

that is, to control of unknown system **optimally**.

Optimal identification for control

$$\theta := (\xi, \delta)$$

$$\widehat{\theta} = (\widehat{\xi}, \widehat{\delta}) \text{ – an estimate}$$

The set of estimates non-falsified by data :

$$\Theta_t := \{ \ \widehat{\theta} \mid |y(\tau) - \widehat{a}y(\tau-1) - \widehat{b}u(\tau-d)| \le \widehat{\delta}_w +$$

$$\widehat{\delta}_y \sup_{\tau-\mu \le s < \tau} |y(\tau)| + \widehat{\delta}_u \sup_{\tau-\mu \le s < \tau} |u(\tau)|, \ \tau = 0, \ldots, t\}$$

– $2(t+1)$ linear in $\widehat{\theta}$ inequalities

The best non-falsified estimate :

$$\theta_t := \operatorname*{argmin}_{\widehat{\theta} \in \Theta_t} \ J_{opt}(\widehat{\theta})$$

$$\Downarrow$$

$$J_{opt}(\theta_t) \le J_{opt}(\theta)$$

since $\theta \in \Theta_t$.

Derivative problems :

1) The computational complexity of the non-convex optimal identification for control.

2) What is the performance of the closed loop system based on the optimal identification?

3) Bounded memory data processing.

Approximately optimal identification

ε_1-grid :

$$\xi_{i,j} = (a_i, b_j)^T := (\,\underline{a} + i\varepsilon_1\,, \underline{b} + j\varepsilon_1\,)^T\,,$$

optimal errors quantification :

$$J_{opt}^{eq}(\widehat{\xi}) := \min_{\{\widehat{\delta}|\ \widehat{\theta}\in\Theta_t\}} \ J_{opt}(\widehat{\xi}, \widehat{\delta})\,,$$

– linear fractional programmings with respect to $\widehat{\delta}$ (linear programmings in $\mathbb{R}^4$).

Approximate solution :

$$\xi_{\varepsilon_1} := \operatorname*{argmin}_{\xi_{i,j}} \ J_{opt}^{eq}(\xi_{i,j})$$

$$J_{opt}(\theta_{\varepsilon_1}) = J_{opt}^{eq}(\xi_{\varepsilon_1})\,.$$

Theorem 2. There exists $C > 0$ such that for any data y_0^t, u_0^{t-1}

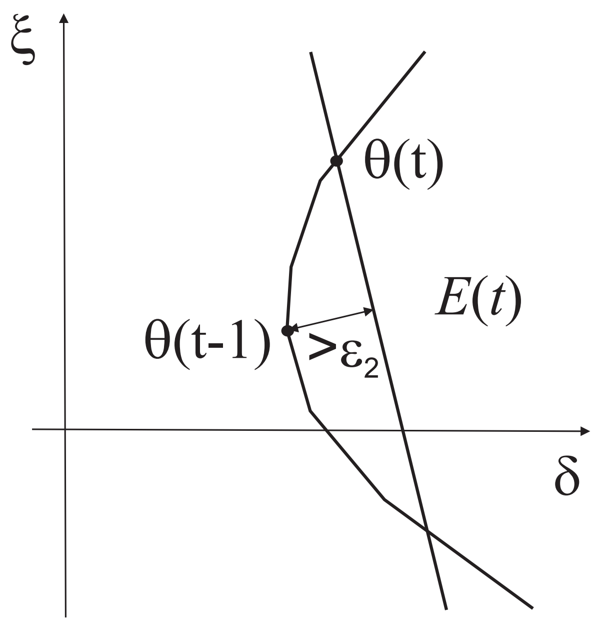
$$J_{opt}(\theta_{\varepsilon_1}) \ \leq \ \min_{\widehat{\theta}\in\Theta_t} \ J_{opt}(\widehat{\theta}) + C\varepsilon_1\,.$$

Optimal steady-state tracking

adaptive controller :

$$K_{opt}(\theta_{\varepsilon_1}(t))$$

identification : $E(t)$ − set estimate



$$\theta_{\varepsilon_1}(t): \quad J_{opt}(\theta_{\varepsilon_1}(t)) \approx \min_{\widehat{\theta}\in E(t)} J_{opt}(\widehat{\theta})$$

Theorem 3. Given

$$\delta_y(1+|a|+\ldots+|a|^{d-1})+\delta_u\frac{|a|^d}{|b|}<1$$

(robust stabilizability of the controlled system), for all sufficiently small $\varepsilon_1>0$ and $\varepsilon_2>0$ final estimates

$$E_\infty := \lim_{t\to\infty} E(t) \quad \text{and} \quad \theta_\infty := \lim_{t\to\infty} \theta(t)$$

are achieved in a finite time and

$$\limsup_{t\to\infty} |\, y(t)-r| \le J_{opt}(\theta)+O(\varepsilon_1)+O(\varepsilon_2)$$

$$(\varepsilon_1,\varepsilon_2\to 0)$$

Example

$$y(t) = 0.98y(t-1) + 0.52u(t-2) + d(t)$$

$$d(t) = 0.1\,\rho_0(t) + 0,04\,\rho_1(t) \max_{t-5\leq<t} |y(s)| +$$

$$0,06\,\rho_2(t) \max_{t-5\leq<t} |u(s)|$$

$\rho_j(t)$ – i.u.d. on [-1,1]

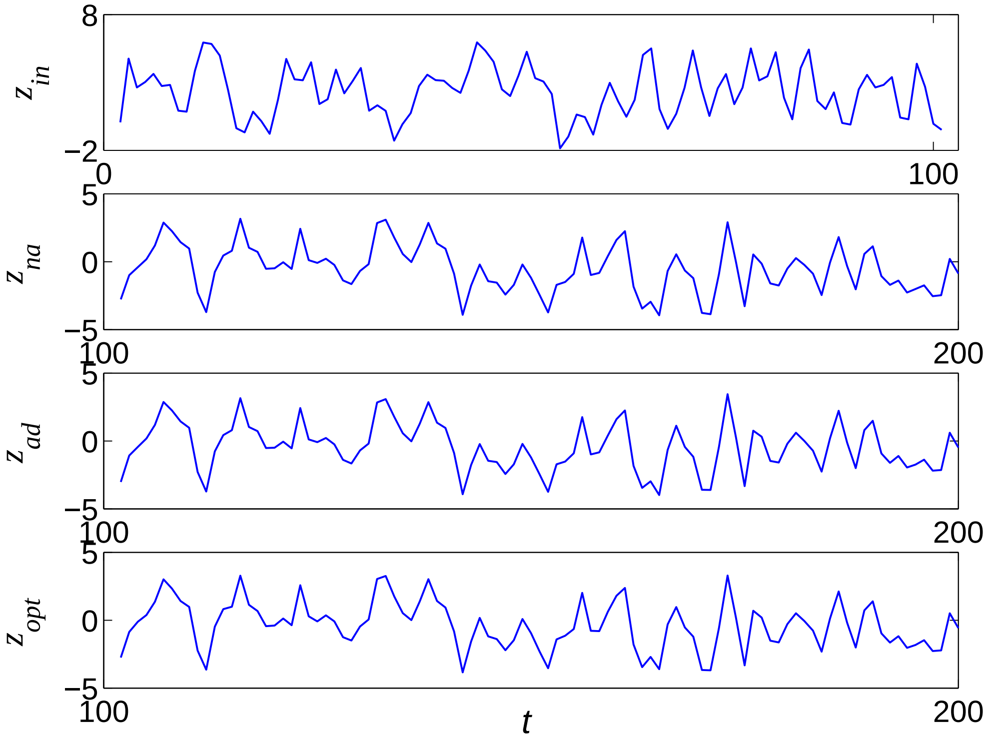
$$\xi \in [\,0.7\,,\,1.2\,] \times [\,0.2\,,\,1\,]$$

The reference signal: $\forall\, t \quad r(t) = 50$.

"Good" initial estimate :

$$\xi(-1) = (\,0.96\,;\,0.48\,)^T$$

Error signals $z(t) = y(t) - r(t)$



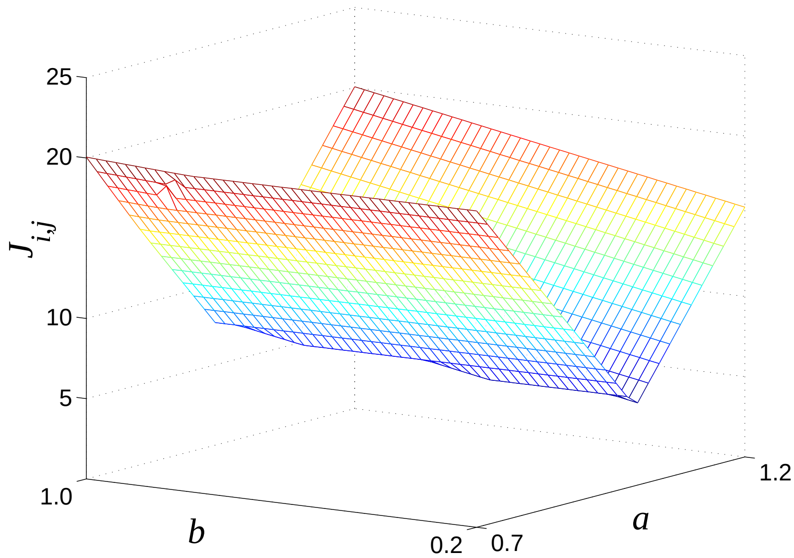
z_{in} – for initial controller;

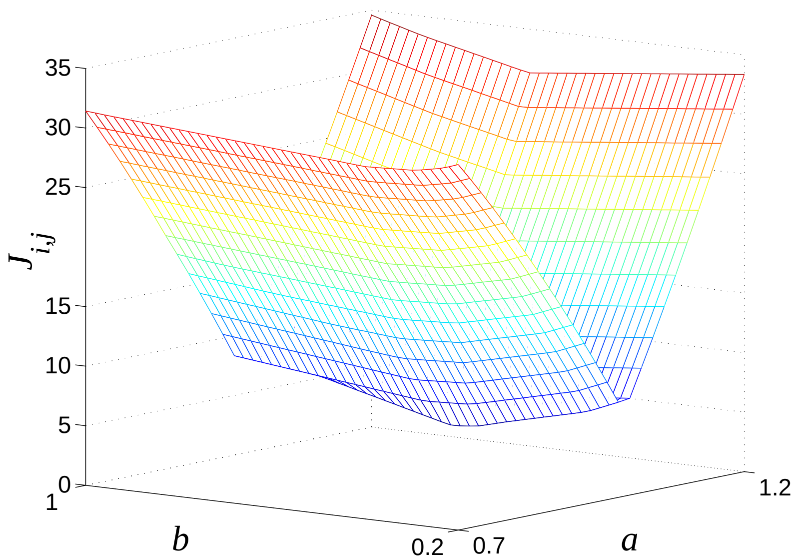
z_{na} – for nonadaptive controller based on $\xi(103)$;

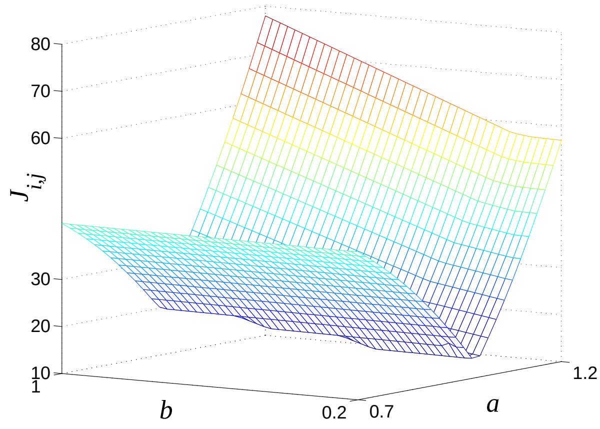
z_{ad} – for adaptive controller;

z_{opt} – for optimal controller for known model.

Optimal identification for data (y_0^{103}, u_0^{102}), $\varepsilon_1 = 0,01$ and $d = 1, 2, 3$.







The minimal values of $J_{opt}(\theta_{\varepsilon_1})$ for $d = 1, 2, 3$ are,respectively, 5.1194, 4.3917, 14.2229.

Then the matlab function *fmincon* was used to compute

$$\theta(103) := (0.9802,\ 0.5549,\ 0,\ 0.0405,\ 0)^T$$

and

$$J(\theta(103)) = J^{eq}(\xi(103) = 4.3639 < 4,4891 =$$

$$J^{eq}(\xi) < J(\theta) = 5.4155 < 6.7577 = J^{eq}(\xi(-1))$$

$$\theta(200) = (0.9797,\ 0.5107,\ 2.1846,\ 0.0023,\ 0)^T$$

$$J(\theta(200)) = 4.9047 < 5,4155 = J(\theta)\,.$$

Conclusion

1. The identification algorithm is adequate to the ℓ_1 robust control theory and involves no other assumptions about uncertainty and exogenous disturbance.

2. All prior information is validated on-line.

3. The optimality of closed loop system is guaranteed. The price for the optimality is in large computations.