CONSTRUCTION OF SPATIAL TRAJECTORIES OF THE UNMANNED FLIGHT VEHICLE

O. G. Andrianova

Bauman Moscow State Technical University, Moscow

Problem statement

The main problem

To construct a spatial trajectory for the flight vehicle with given boundary conditions subject to the state and control constraints.

Special problems

1. To find a trajectory for an altitude shift case, when flight time is unknown
2. To obtain flight time dependence on the altitude (the set of typical maneuvers)

Mathematical model

$$\dot V = (n_x - \sin\theta)\, g, \qquad \dot H = V \sin\theta,$$
$$\dot\theta = \frac{(n_y \cos\gamma - \cos\theta)\, g}{V}, \qquad \dot L = V \cos\theta \cos\psi, \qquad (1)$$
$$\dot\psi = -\frac{n_y\, g \sin\gamma}{V \cos\theta}, \qquad \dot Z = -V \cos\theta \sin\psi.$$

Conventional signs

V — velocity, m/sec;
H — altitude, m;
θ — flight path angle, rad;
L — longitudinal distance, m;
ψ — course angle, rad;
Z — lateral distance, m;

n_x — longitudinal load factor;
n_y — transverse load factor;
γ — roll angle, rad;
g — sea-level acceleration of gravity, m/sec^2.

Control variables: $U = (n_x, n_y, \gamma)$;
State variables: $X = (V, \theta, \psi, H, L, Z)$.

State and control constraints

$$D_x = \Big\{ V \in [V_{min}, V_{max}],\ \ |\theta| < \frac{\pi}{2},\ \ \psi \in [\psi_{min}, \psi_{max}],$$
$$H \in [H_{min}, H_{max}],\ \ L \in [L_{min}, L_{max}],\ \ Z \in [Z_{min}, Z_{max}] \Big\}, \quad (2)$$

$$D_u = \{ |\gamma| < \gamma_{max},\ \ n_x \in [n_{x,min}, n_{x,max}],\ \ n_y \in [n_{y,min}, n_{y,max}] \}. \quad (3)$$

Canonical form transformation

New virtual control variables: $v_1 = n_x$, $v_2 = n_y \cos\gamma$, $v_3 = n_y \sin\gamma$.
Define new state variables $y_1 = H$, $y_2 = L$, $y_3 = Z$.
The mapping $\Phi(X) = (y_1, \dot{y}_1, y_2, \dot{y}_2, y_3, \dot{y}_3)$ is invertible ($\exists \Phi^{-1}$) in the domain (2),(3).

The system in canonical form

$$\begin{aligned}
\ddot{y}_1 &= -g + v_1 g \sin\theta + v_2 g \cos\theta, \\
\ddot{y}_2 &= v_1 g \cos\theta \cos\psi - v_2 g \sin\theta \cos\psi + v_3 g \sin\psi, \\
\ddot{y}_3 &= -v_1 g \cos\theta \sin\psi + v_2 g \sin\theta \sin\psi + v_3 g \cos\psi.
\end{aligned} \quad (4)$$

Altitude shift

Mathematical model

Movement in the vertical plane is described by equations

$$Z = const; \quad \dot{Z} = 0; \quad \psi = 0; \quad \dot{\psi} = 0; \quad \gamma = 0 \quad \Longrightarrow$$

$$\begin{aligned} \dot{V} &= (n_x - \sin\theta)\, g, & \dot{H} &= V \sin\theta, \\ \dot{\theta} &= \frac{(n_y - \cos\theta)\, g}{V}, & \dot{L} &= V \cos\theta. \end{aligned} \tag{5}$$

Boundary conditions

$$t = t_0 \quad \Longrightarrow \quad \begin{aligned} &V_0 = V^*; & &H_0 = H^*; \\ &\theta_0 = 0; & &L_0 = 0; \\ &n_{x_0} = 0; & &n_{y_0} = 1; \end{aligned} \qquad t = t_1 \quad \Longrightarrow \quad \begin{aligned} &V_1 = V^*; & &H_1 = H^* + \Delta H; \\ &\theta_1 = 0; & &L_1 = L^*; \\ &n_{x_1} = 0; & &n_{y_1} = 1; \end{aligned}$$

t_1, L_1 should be found.

The system in canonical form

Canonical variables: $y_1 = H, y_2 = L$.

$$\ddot{y}_1 = -g + v_1\, g \sin\theta + v_2\, g \cos\theta, \qquad (6)$$
$$\ddot{y}_2 = v_1\, g \cos\theta - v_2\, g \sin\theta.$$

Planned trajectory as a time function

$$y_1(t) = H(t) = H^* + \frac{10\,\Delta H}{T^3}(t-t_0)^3 - \frac{15\,\Delta H}{T^4}(t-t_0)^4 + \frac{16\,\Delta H}{T^5}(t-t_0)^5, \qquad (7)$$
$$y_2(t) = L(t) = V^*(t-t_0) + \frac{10\,(L^* - V^*T)}{T^3}(t-t_0)^3 - \frac{15\,(L^* - V^*T)}{T^4}(t-t_0)^4 +$$
$$+ \frac{16\,(L^* - V^*T)}{T^5}(t-t_0)^5,$$

where $T = t_1 - t_0$.

Search algorithm for t_1 and L_1

1. Define $L_{10} = H^*$, $t_{10} = \frac{H^*}{V_{max}}$;
2. Construct a planned trajectory;
3. Check, whether control and state constraints are satisfied;
4. If not, increase t_1 and L_1.

Time t_1 selection criterion

$$\Delta V \longrightarrow \min, \quad \Delta V = max(V) - min(V), \tag{8}$$

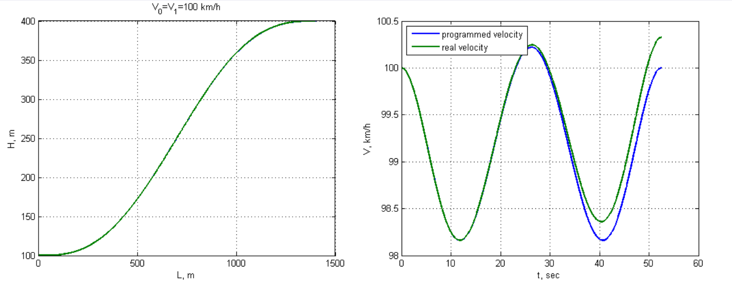
and $X \in D_x, \quad U \in D_u$.

Programmed control as a time function

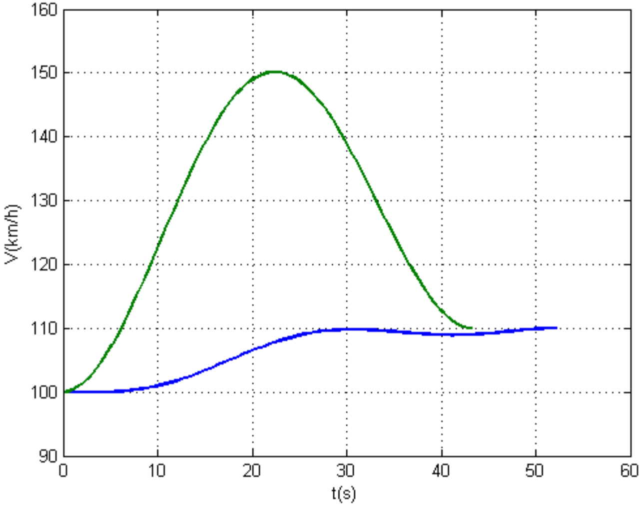
$$v_1(t) = \frac{(\ddot{y}_1+g)\sin\theta+\ddot{y}_2\cos\theta}{g},$$
$$v_2(t) = \frac{(\ddot{y}_1+g)\cos\theta-\ddot{y}_2\sin\theta}{g}. \tag{9}$$

Flight path angle estimation

$$\sin\theta = \frac{\dot{y}_1}{\sqrt{\dot{y}_1^2+\dot{y}_2^2}},$$
$$\cos\theta = \frac{|\dot{y}_2|}{\sqrt{\dot{y}_1^2+\dot{y}_2^2}}. \tag{10}$$



Speed deviation



Path planning for altitude shift subject to given longitudinal distance

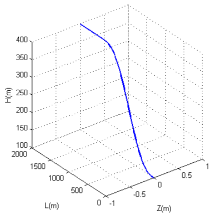
$L^* < L_{max}$, the motion from $(H_1, L_1, Z_1) = (H^*, L^*, C)$ to $(H_2, L_2, Z_2) = (H^*, L_{max}, C)$ is constant, where $C = const$.

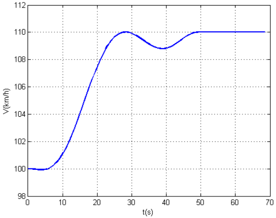
The time of constant motion is defined as $\Delta t = \frac{L_{max} - L^*}{V^*}$.

$$\forall t \in [t_1, t_1 + \Delta t] \qquad \begin{array}{l} y_1(t) = H^*, \;\; \dot{y}_1(t) = \ddot{y}_1(t) = 0; \\ y_2(t) = L^* + V^*(t - t_1), \;\; \dot{y}_2(t) = V^*, \;\; \ddot{y}_2(t) = 0. \end{array} \qquad (11)$$

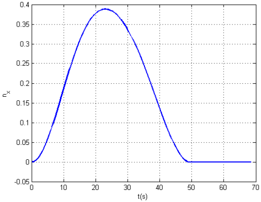
Example of planned trajectory construction

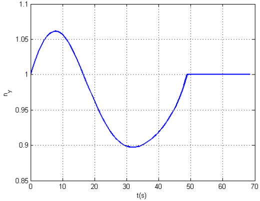
Planned trajectory and velocity





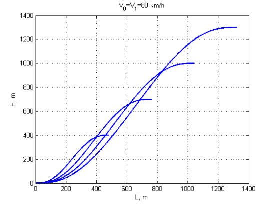
Programmed control



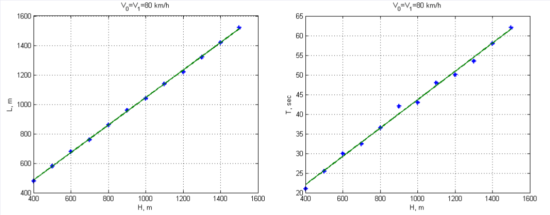


Set of typical maneuvers example

Planned trajectories



Flight time and longitudinal distance depending on the altitude



Conclusions

1. The equations for planned trajectory construction in case of altitude shift are given;
2. The search algorithm for flight time and longitudinal distance is worked out;
3. Flight time dependence on the altitude is obtained for specified boundary velocities;
4. The developed set of typical maneuvers allows to define the flight time for any possible altitude without restarting search algorithm.

Thank you!