# Algorithm of bench accelerometer calibration based on the guaranteed estimation approach

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# Model of accelerometer unit

**Instrumental coordinate system:**

$$Op_1 p_2 p_3 \qquad \cos\left(a_i,\, p_j\right) = \mu_{ij}$$

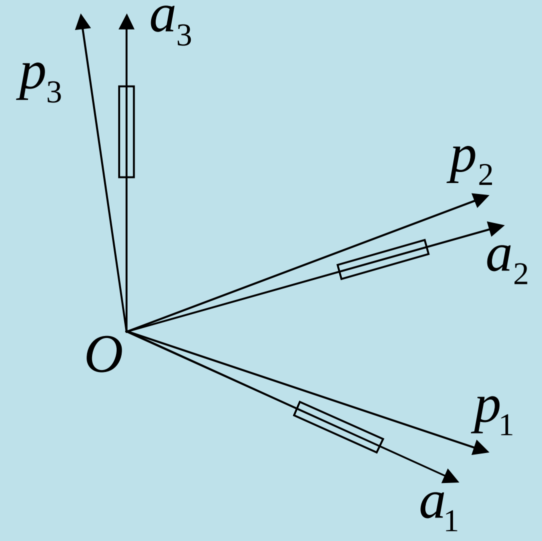
**Accelerometer output data:**

$$\hat{f}_i = (1 + k_i)\, f_{ai} + \varepsilon_i + \delta f_i$$

$$f = a - g^0, \qquad |\,\delta f_i\,| \le \delta f_{\max}$$

**Matrix model of unit:**

$$\hat{f} = (E + C)\, f_p + \varepsilon + \delta f$$




Accelerometer unit
and instrumental axes

$$\hat{f} = \begin{pmatrix} \hat{f}_1 \\ \hat{f}_2 \\ \hat{f}_3 \end{pmatrix}, \qquad C \approx \begin{pmatrix} k_1 & \mu_{12} & \mu_{13} \\ \mu_{21} & k_2 & \mu_{23} \\ \mu_{31} & \mu_{32} & k_3 \end{pmatrix}, \qquad \varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix}, \qquad \delta f = \begin{pmatrix} \delta f_1 \\ \delta f_2 \\ \delta f_3 \end{pmatrix}$$

## Main calibration equation

**Information about unit orientation:** $n = \left(n_1,\ n_2,\ n_3\right)^T$

**Main calibration equation:**

$$n^T \frac{\hat{f}}{\hat{g}} - 1 = n^T C^0 n + \frac{n^T \varepsilon}{\hat{g}} + r(n), \qquad C^0 = C - \frac{\Delta g}{\hat{g}}$$

## Standard form of estimation problem

$$z(n) = H(n)q + r(n), \qquad |\, r(n) \,| \leq r_{\max}(n)$$

$$z(n) = n^T \frac{\hat{f}}{\hat{g}} - 1,$$

$$H(n) = (n_1^2, \quad n_2^2, \quad n_3^2, \quad n_1 n_2, \quad n_1 n_3, \quad n_2 n_3, \quad n_1, \quad n_2, \quad n_3),$$

$$q = \left( c_{11}^0,\ c_{22}^0,\ c_{33}^0,\ c_{12}^0 + c_{21}^0,\ c_{13}^0 + c_{31}^0,\ c_{23}^0 + c_{32}^0,\ \frac{\varepsilon_1}{\hat{g}},\ \frac{\varepsilon_2}{\hat{g}},\ \frac{\varepsilon_3}{\hat{g}} \right)^T$$

# Calibration extremal problem

## Extremal problem:

$$z(n) = H(n)q + r(n), \qquad \| n \| = 1, \qquad | r(n) | \le r_{\max}(n)$$

$J = a^T q$ (напр., для оценивания $q_1$ $\quad a = (1, 0, 0, \ldots, 0)^T$ )

$$\tilde{J} = \int\limits_{\|n\|=1} \Phi_0(n) z(n) dn + \sum_{s=1}^{M} \Phi_s z(n_s) = \int\limits_{\|n\|=1} \Phi(n) z(n) dn,$$

$$\Phi(n) = \Phi_0(n) + \sum_{s=1}^{M} \Phi_s \delta(n - n_s)$$

$$\sup_{q,\rho} | \tilde{J} - J | \to \min_{\Phi}$$

## Variational problem (moment problem):

$$\int\limits_{\|n\|=1} r_{\max}(n) | \Phi(n) | dn \to \inf_{\Phi},$$

$$\int\limits_{\|n\|=1} H^T(n) \Phi(n) dn = a.$$

## Theorems on optimal estimator structure

1. There exists at least one solution $\Phi(n)$ such that $\Phi(n) \neq 0$ at $m$ orientation vectors $n$; here $m \leq \dim q$.
2. Optimal values for the primal problem and for the dual problem

$$I^0 = \sup_{\lambda \in R^m} a^T \lambda, \qquad |H(n)\lambda| \leq 1, \quad \|n\| = 1$$

are equal.

3. The set of optimal measurement points consists only of extremal points for the generalized Chebyshev polynomial:

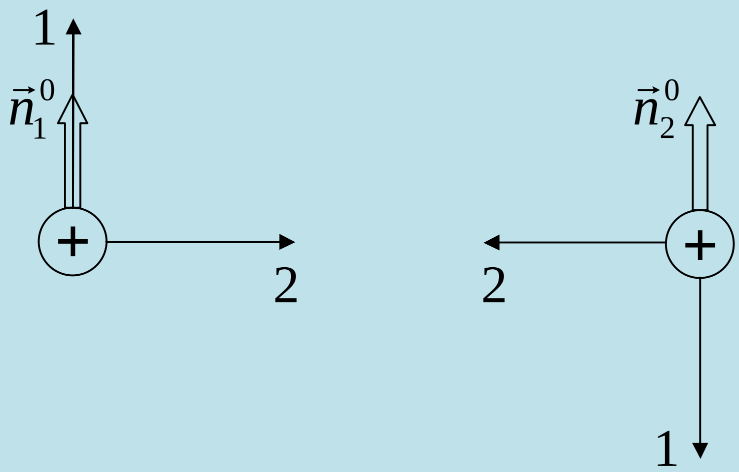
$$\max_{\|n\|=1} \left| H_1(n) + \sum_{i=2}^{9} y_i H_i(n) \right| \to \min_{y_2,\ldots y_9}$$

Estimator components are determined by the equalities

$$\int_{\|n\|=1} H^T(n)\Phi(n)dn = a \quad \to \quad \sum_{s=1}^{d} H^T(n_s)\Phi_s^0 = a$$

# Optimal design of experiment

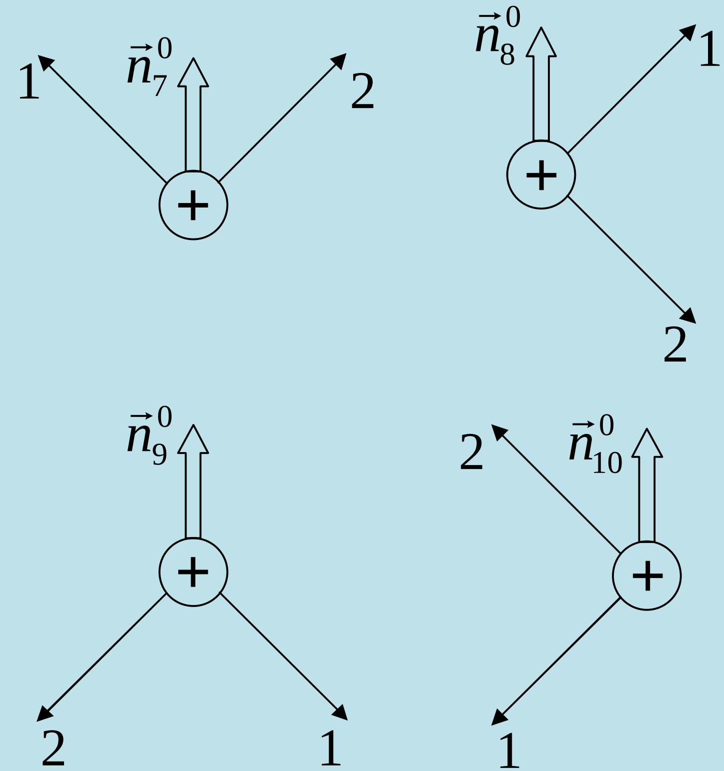
$$q = \left(q_1, \quad q_2, \quad \ldots, \quad q_9\right)^T$$




$$\tilde{q}_1 = \frac{z(n_{(1)}^0) + z(n_{(2)}^0)}{2}$$

$$\tilde{q}_7 = \frac{z(n_{(1)}^0) - z(n_{(2)}^0)}{2}$$

Orientations for biases and scale coefficients estimation




$$\tilde{q}_4 = \frac{z(n_{(7)}^0) - z(n_{(8)}^0) + z(n_{(9)}^0) - z(n_{(10)}^0)}{2}$$

Orientations for axes misalignments estimation

## Second iteration

**Modified instrumental coordinate system:** $Op_1^* p_2^* p_3^*$.
In this system $C = C^T$.

$$C = C^{(1)} + \Delta C^{(1)}, \qquad \frac{\varepsilon}{\hat{g}} = \left(\frac{\varepsilon}{\hat{g}}\right)^{(1)} + \Delta\left(\frac{\varepsilon}{\hat{g}}\right)^{(1)}$$

## Modified main calibration equation

$$\hat{f} = (E + C) f_p + \varepsilon + \delta f$$

$$n^T \left(\frac{\hat{f}}{\hat{g}}\right)^{(2)} - 1 = n^T C_0^{(2)} n + n^T \left(\frac{\varepsilon}{\hat{g}}\right)^{(2)} + r^{(2)}(n)$$

$$\left(\frac{\hat{f}}{\hat{g}}\right)^{(2)} = \left(E + C^{(1)}\right)^{-1} \left(\frac{\hat{f}}{\hat{g}} - \left(\frac{\varepsilon}{\hat{g}}\right)^{(1)}\right), \qquad \left(\frac{\varepsilon}{\hat{g}}\right)^{(2)} = \left(E + C^{(1)}\right)^{-1} \Delta\left(\frac{\varepsilon}{\hat{g}}\right)^{(1)},$$

$$C^{(2)} = \left(E + C^{(1)}\right)^{-1} \Delta C^{(1)}, \qquad C_0^{(2)} = C^{(2)} - \frac{\Delta g}{\hat{g}} E$$

# Simulation results

## Example 1:

$$\vartheta_i^{(s)} = 2\cdot 10^{-2}$$

$$\varphi_i^{(s)} = -2\cdot 10^{-2} \quad (s=1)$$

$$\varphi_i^{(s)} = 2\cdot 10^{-2} \quad (s = 2, 3, \ldots, 18)$$

$$\psi_i = 0$$

$$c_{11} = c_{22} = c_{33} = c_{12} = c_{23} = 2\cdot 10^{-2}$$

$$c_{13} = -2\cdot 10^{-2}$$

$$\Delta g = 10^{-4}\,\hat{g}$$

$$\varepsilon_i = 10^{-2}\,\hat{g}$$

$$\delta f_i^{(s)} = -10^{-4}\,\hat{g}$$

| Parameter | 1st iteration error | 2nd iteration error | Errors ratio |
|---|---|---|---|
| $c_{11}$ | $4{,}8\cdot 10^{-4}$ | $1{,}1\cdot 10^{-4}$ | 4,5 |
| $c_{22}$ | $5{,}0\cdot 10^{-4}$ | $1{,}1\cdot 10^{-4}$ | 4,6 |
| $c_{33}$ | $2{,}9\cdot 10^{-4}$ | $1{,}0\cdot 10^{-4}$ | 2,8 |
| $c_{12}$ | $5{,}9\cdot 10^{-4}$ | $2{,}0\cdot 10^{-4}$ | 3,0 |
| $\varepsilon_1 / \hat{g}$ | $9{,}2\cdot 10^{-4}$ | $9{,}5\cdot 10^{-5}$ | 9,8 |

# Main results

General iteration accelerometers bench calibration methodology is presented. It is based on the guaranteed estimation approach. The methodology does not require high accuracy of unit orientation.

- Calibration problem solution reduced to the spatial guaranteed estimation problem solution.
- Optimal experiment design described.
- Expressions for estimation accuracy are obtained.
- Second iteration is investigated in detail. This iteration significantly improves estimation accuracy.

# Averaged statistical analysys:

$$e_2 = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\tilde{p}_i - p_i\right)^2}$$

| Parameter | $e_2$ (1st iteration) | $e_2$ (2nd iteration) | Errors $e_2$ ratio |
|---|---|---|---|
| $c_{11}$, $c_{22}$, $c_{33}$ | $2{,}7\cdot10^{-4}$ | $1{,}6\cdot10^{-4}$ | 1,7 |
| $c_{12}$, $c_{13}$, $c_{23}$ | $1{,}4\cdot10^{-4}$ | $9{,}1\cdot10^{-5}$ | 1,5 |
| $\varepsilon_1/\hat{g}$, $\varepsilon_2/\hat{g}$, $\varepsilon_3/\hat{g}$ | $1{,}8\cdot10^{-4}$ | $1{,}2\cdot10^{-4}$ | 1,5 |