On the traffic management tools in BMW and Stable dynamic equilibrium models

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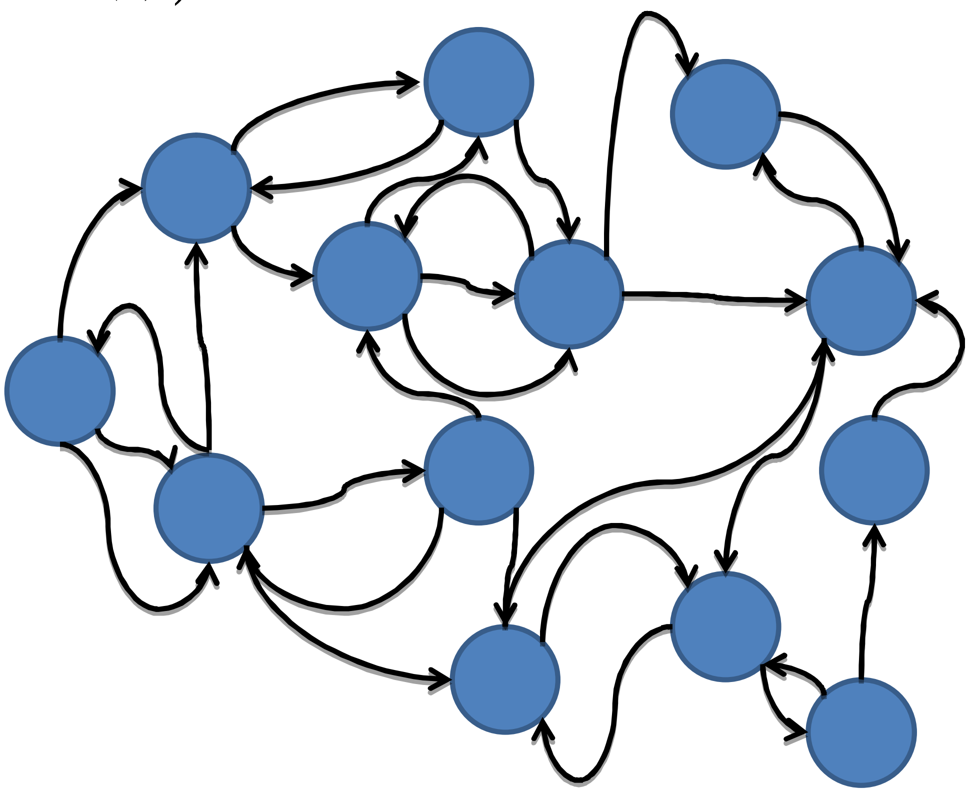
PreMoLab

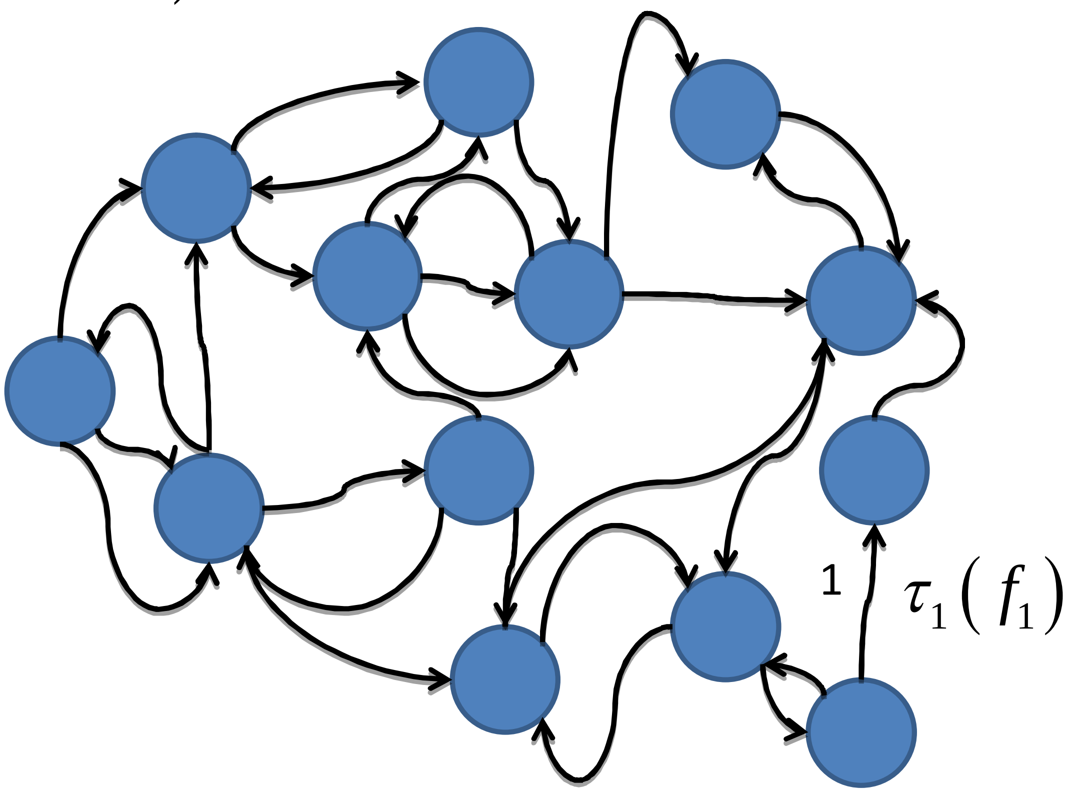
Intro

$$\Gamma\left(V, E, \tau\left(\cdot\right)\right)$$

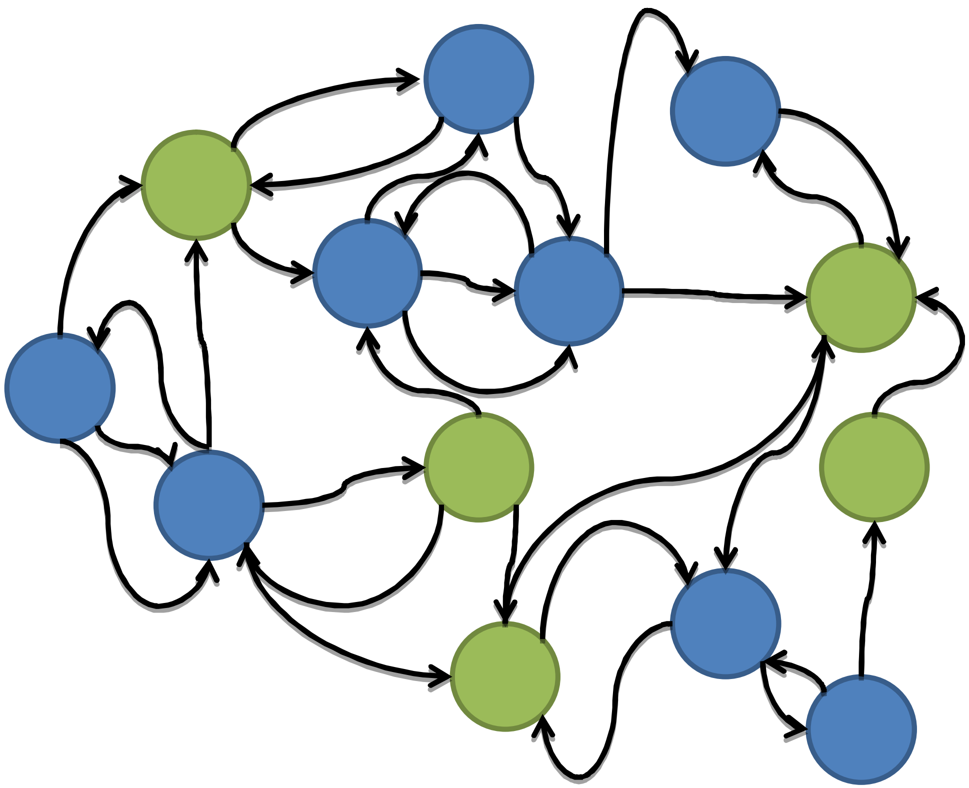


Intro

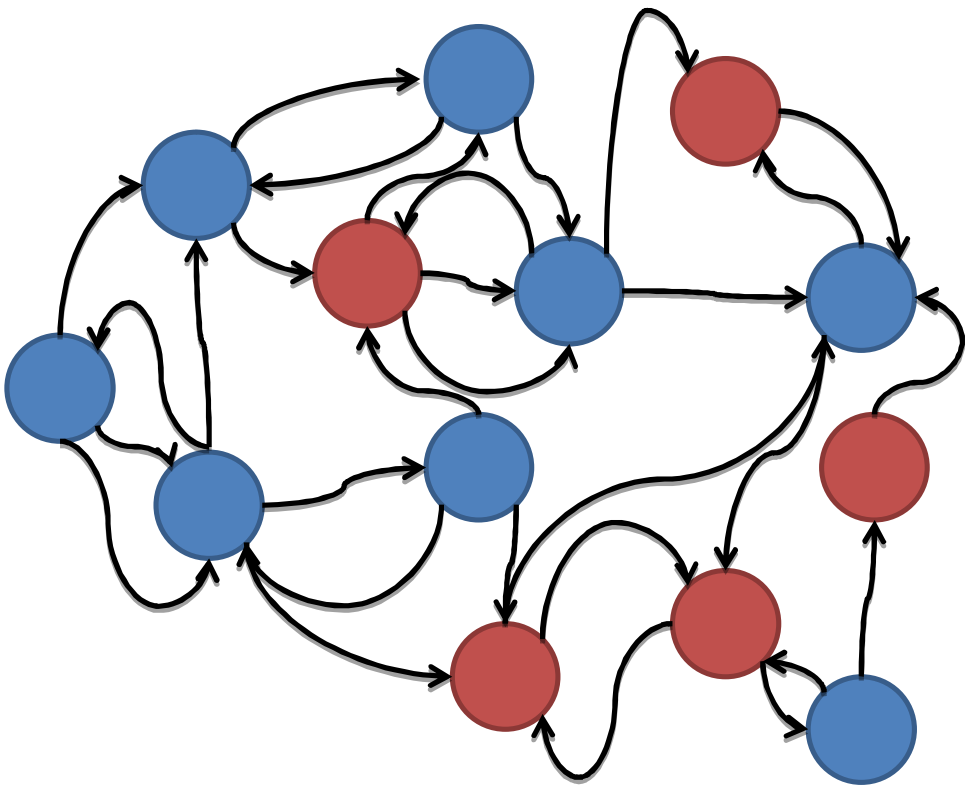
$$\Gamma\left(V, E, \vec{\tau}(\cdot)\right)$$



Set of origin nodes

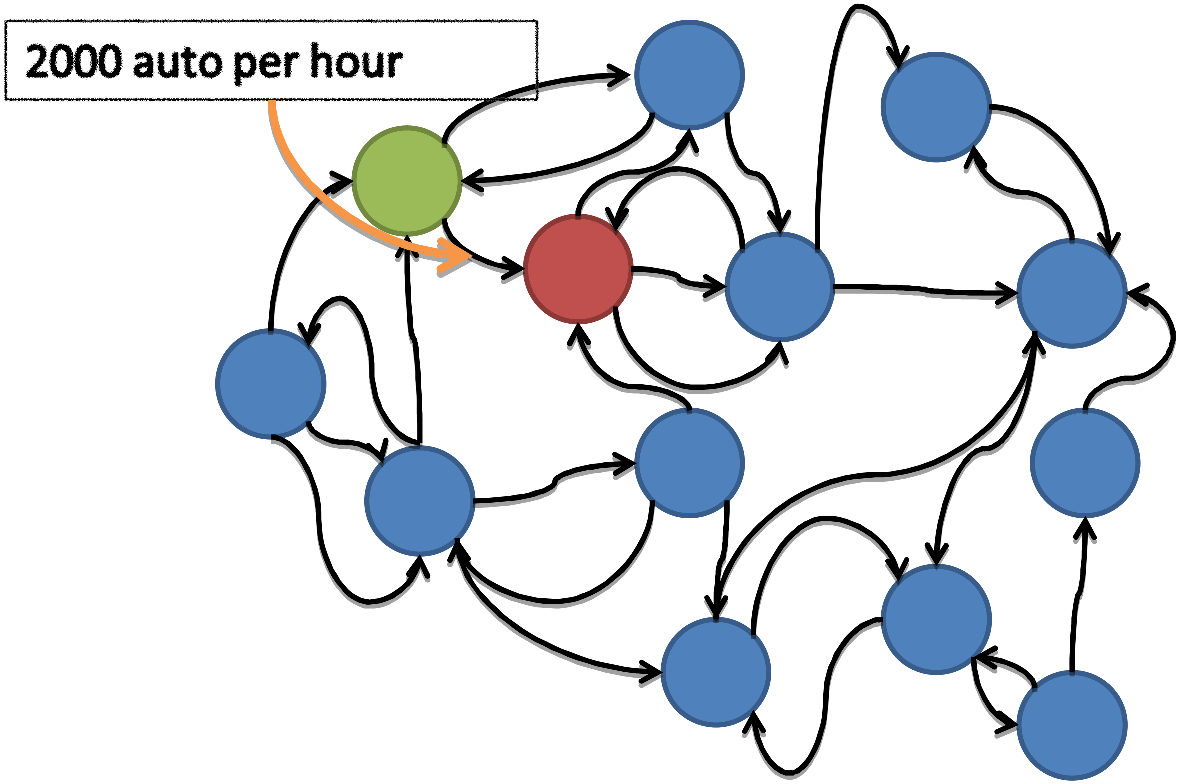


Set of destination nodes

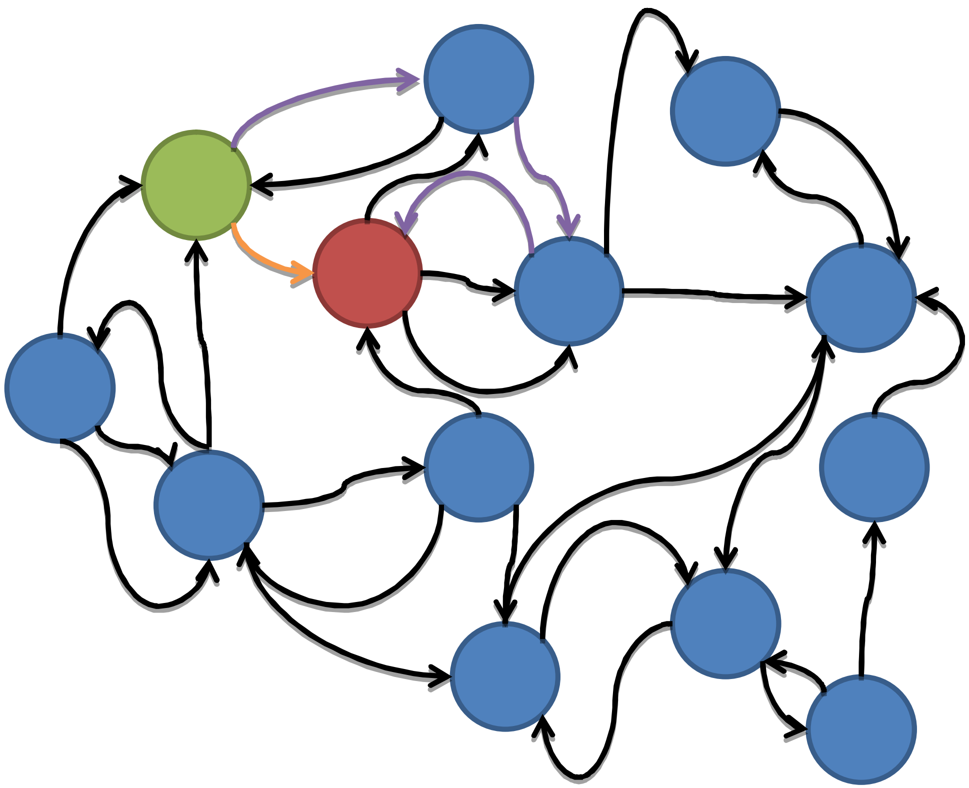


Intro

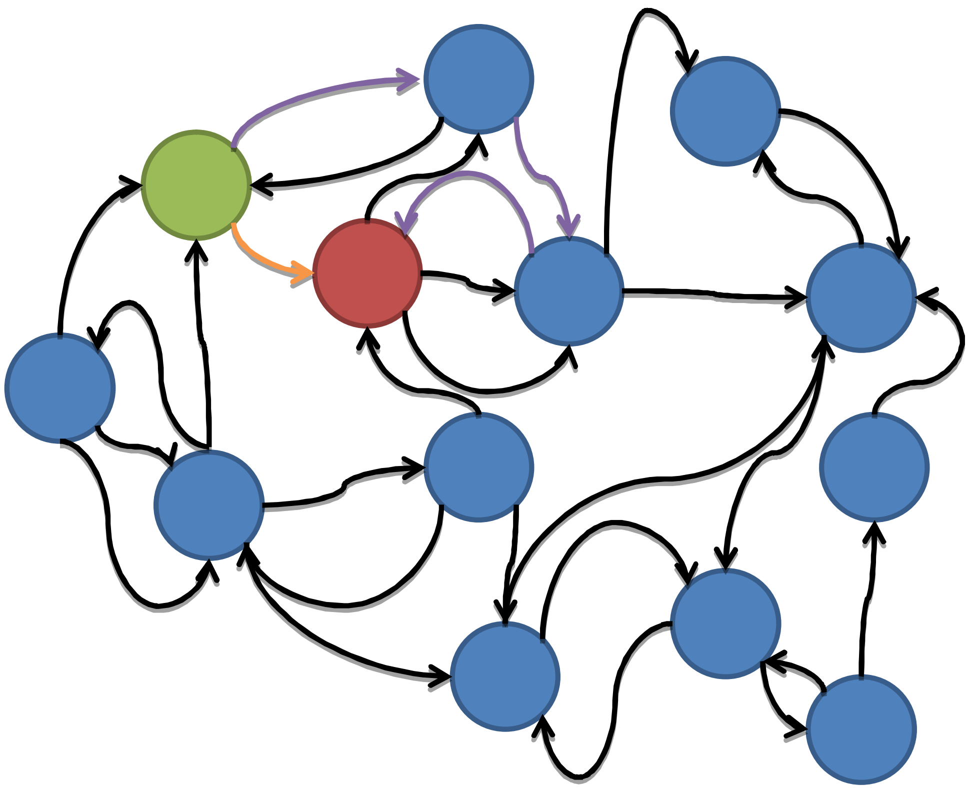
Each origin-destination pair corresponds to traffic flow between them



Intro



Drivers are greedy! Only the cheapest paths are used.



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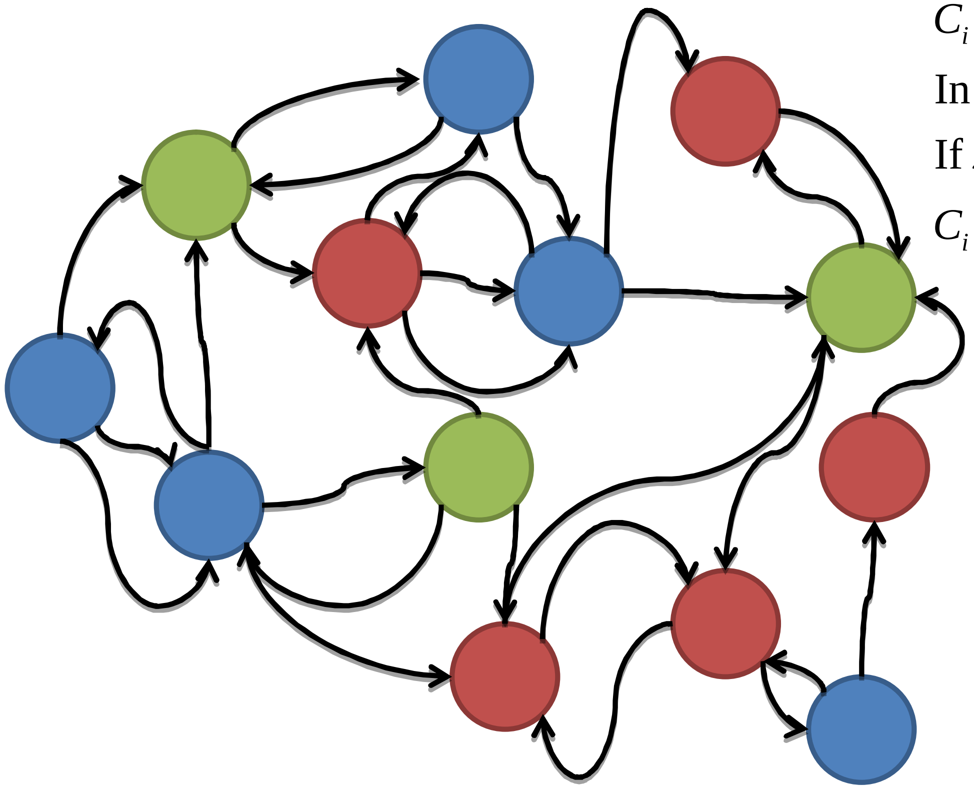
Intro



$\vec{x}$ – distribution of traffic flows by paths

$C_i(\vec{x})$ – cost of i-th path at state $\vec{x}$

In equilibrium the following statement holds:

If $x_i > 0$ (i-th path corresponds to ω OD-pair), then

$$C_i(\vec{x}) = \min_{q \in P_w} C_q(\vec{x})$$

Intro

Definition

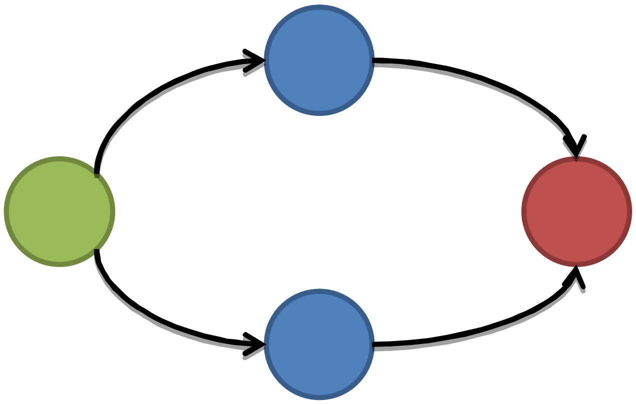
$\vec{x}^{opt}$ is a social optimum, if

$$\vec{x}^{opt} = \arg\min_{\vec{x}\in X}\left(\sum_{i\in P} C_i\left(\vec{x}\right)x_i\right)$$

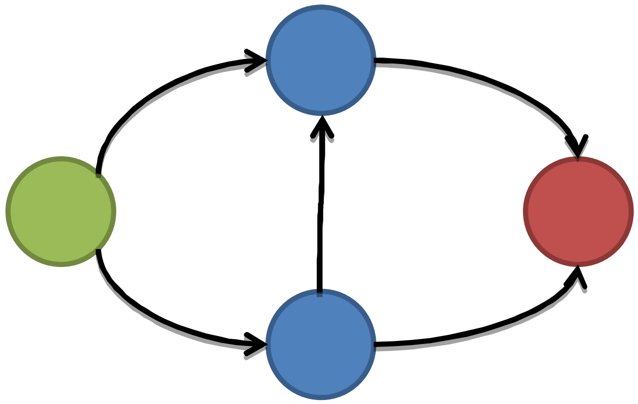
Problem:

$$\vec{x}^{opt} \neq \vec{x}^{eq}$$

Special case is Braess Paradox



Can be better, than



Inefficient arcs

One can manage problem with inefficiency at $\vec{x}^{eq}$ using road tolls

$$\tau_e\left(\vec{x}\right) \to \tau_e\left(\vec{x}\right) + \tau_e^c$$

τ_e^c is const

$$\vec{x}^{eq} \to \vec{x}^{eq}\left(\vec{\tau}^{\,c}\right)$$

We need to find $\vec{\tau}^{\,c}, s.t.$

$$\vec{\tau}^{\,opt} = \arg\min_{\vec{\tau}^{\,c}} \left(\sum_{i \in P} C_i\left(\vec{x}^{eq}\left(\vec{\tau}^{\,c}\right)\right) x_i^{eq}\left(\vec{\tau}^{\,c}\right) \right)$$

or s.t. $\vec{x}^{eq}\left(\vec{\tau}^{\,c}\right) = \vec{x}^{opt}$

We know, that in our models such $\vec{\tau}^{\,c}$ always exists.

Find $\vec{\tau}^{\,c}$ is a hard problem.

Main problem: inefficiency of arcs depends on the current state $\vec{x}$

Inefficient arcs

Note 1:

If $x_i^{eq}\left(\vec{\tau}^{\,opt}\right)=0$, then if $\tau_i^{opt}\to\tau_i^{opt}+\varepsilon,\ \varepsilon\geq 0$

equilibrium will be the same.

Note 2:

$$\min_{\vec{\tau}^{\,c}}\left(\sum_{i\in P}C_i\left(\vec{x}^{eq}\left(\vec{\tau}^{\,c}\right)\right)x_i^{eq}\left(\vec{\tau}^{\,c}\right)\right)\geq\sum_{i\in P}C_i\left(\vec{x}^{opt}\right)x_i^{opt}$$

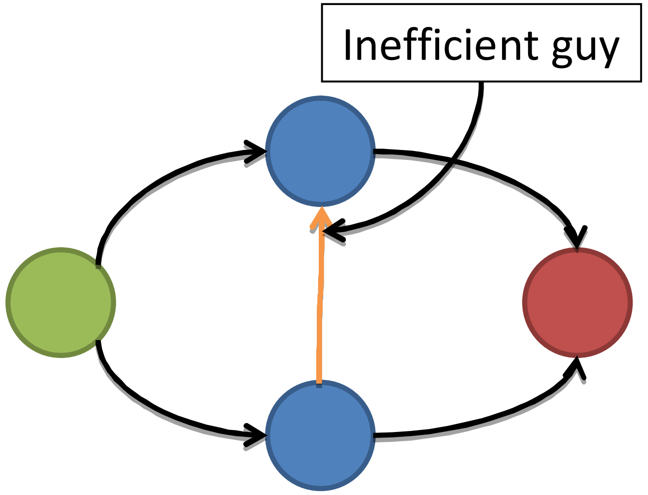
Scheme

Find i-th arc, s.t. $y_i^{opt}=0$

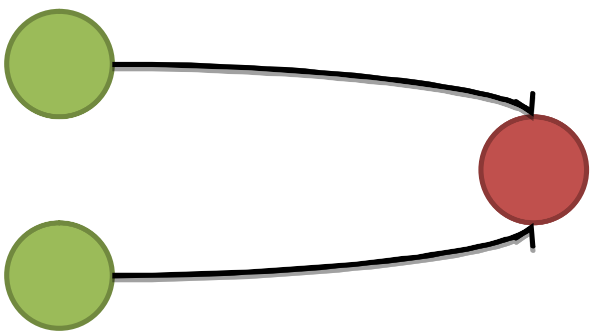
$$y_i^{opt}=\sum_{j\in P}x_j\delta_{ij}$$

i-th arc is a good candidate!

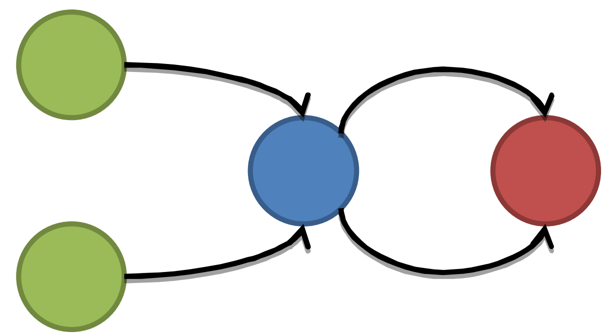
Why? Becouse this guy is always inefficient !



Inefficient nodes

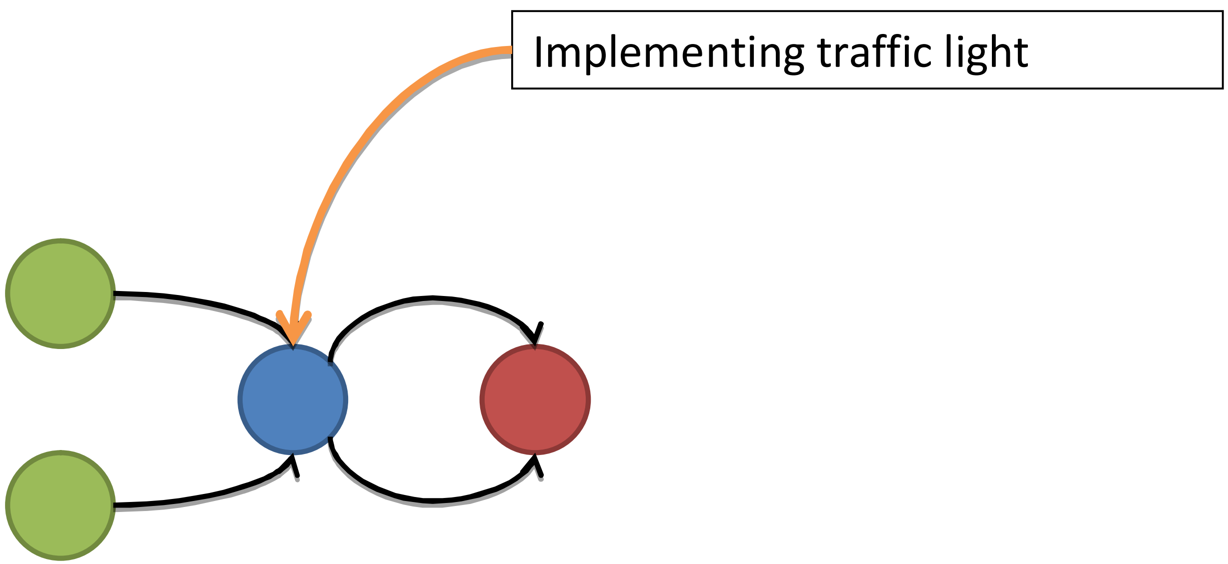


Can be better, than

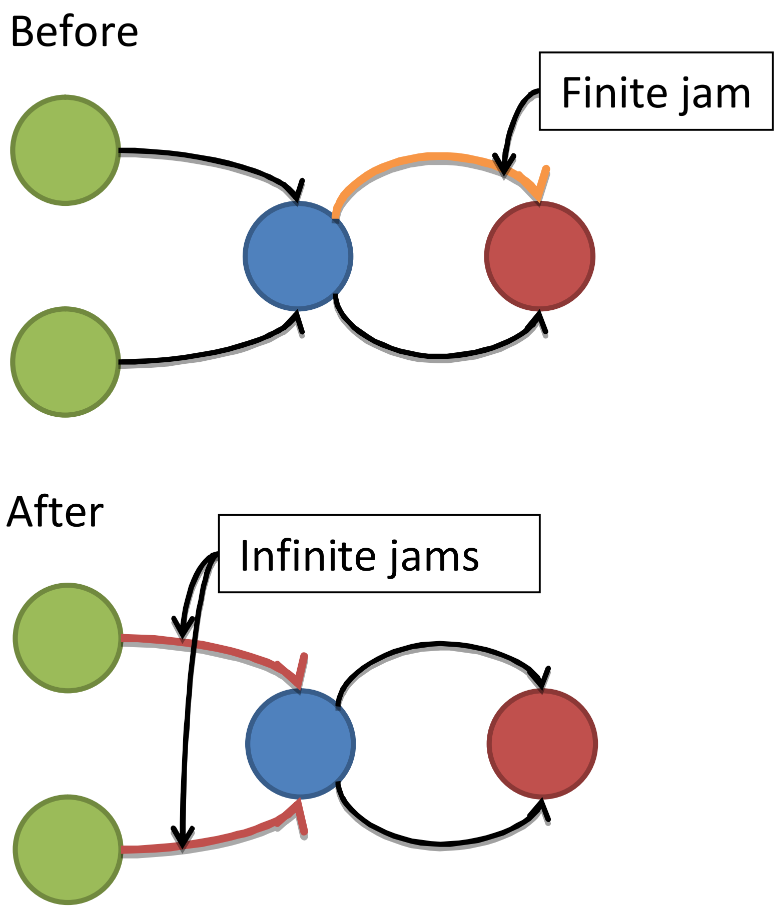


Inefficient nodes

Intuitive strategy: by using traffic light we reduce traffic at the short arc.



Inefficient nodes



Inefficient nodes

Intuitive strategy fails!

Implementing traffic light can make things get worse!