STABILIZATION OF ORIENTATION OF A SATELLITE WITH AN ELASTIC ROD

Anastasia S. Khalina

Moscow Aviation Institute
(National Research University)

June 2012

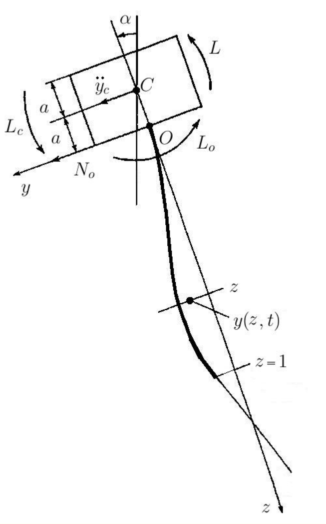

Figure : Satellite.

J_c – moment of inertia;
L – disturbing moment;
N_o – force of reaction of the rod in the point O;
L_o – moment of force of reaction of the rod in the point O;
y_c – displacement of the center of mass.

Oyz – is a sistem of coordinates connected with the satellite C,
α – angle from the local vertical, i.e. a mistake of stabilizing system.

2. A satellite movement model. Mathematical model

Equations of disturbed motion of the satellite

$$J_c\ddot{\alpha} = u - \Omega\alpha + aN_0 + L_0, \qquad L_0 = -\frac{EJ}{l^2}\left(1 + h\frac{\partial}{\partial t}\right)\frac{\partial^2 y(0,t)}{\partial z^2},$$
$$m_c\ddot{y}_c = N_0, \qquad N_0 = -\frac{EJ}{l^3}\left(1 + h\frac{\partial}{\partial t}\right)\frac{\partial^3 y(0,t)}{\partial z^3}. \qquad (1)$$

The equation describing elasticity of the rod look like

$$\frac{\partial^2 y}{\partial t^2} + \frac{EJ}{m_z l^2}\left(1 + h\frac{\partial}{\partial t}\right)\frac{\partial^4 y}{\partial z^4} = -\ddot{y}_c + (a + lz)\ddot{\alpha}, \qquad (2)$$

here EJ – rigidity of the rod, m_z – running weight of the rod.

Boundary conditions for the flexure $y(z, t)$ look like

$$y(0,t) = 0, \qquad \frac{\partial y(0,t)}{\partial z} = 0,$$
$$\frac{\partial^2 y(1,t)}{\partial z^2} = 0, \qquad \frac{\partial^3 y(1,t)}{\partial z^3} = 0. \qquad (3)$$

3. A satellite movement model. A special case. Rigid satellite

Equations of motion of the rigid satellite

$$J_c\ddot{\alpha} = u - \Omega\alpha. \tag{4}$$

Control look like

$$u = -\phi_1\alpha - \phi_2\dot{\alpha}. \tag{5}$$

$$\phi_1 + \Omega > 0, \qquad \phi_2 > 0. \tag{6}$$

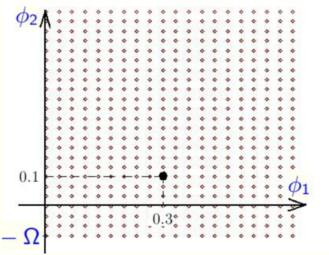

Figure : Stability diagram.

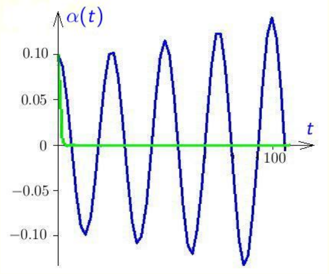

Figure : Graph of $\alpha(t)$.

The equation describing elasticity of the rod look like

$$\frac{\partial^2 y}{\partial t^2} + \frac{EJ}{m_z l^2}(1 + h\frac{\partial}{\partial t})\frac{\partial^4 y}{\partial z^4} = -\ddot{y}_c + (a + lz)\ddot{\alpha}.$$

Boundary conditions for the flexure $y(z, t)$ look like

$$\begin{array}{ll} y(0,t) = 0, & \frac{\partial y(0,t)}{\partial z} = 0, \\ \frac{\partial^2 y(1,t)}{\partial z^2} = 0, & \frac{\partial^3 y(1,t)}{\partial z^3} = 0. \end{array}$$

Solution $y(z, t)$ look like

$$y(z,t) = y_0(z,t) + \Delta y(z,t). \tag{7}$$

5. Quasi-static component of the solution

The equation describing elasticity of the rod for quasi-static component $y_0(z,t)$ of the solution (7) look like

$$\frac{EJ}{m_z l^3}\frac{\partial^4 y_0(z,t)}{\partial z^4} = -\ddot{y}_c + (a + lz)\ddot{\alpha} \qquad (8)$$

with boundary conditions (3).
The equation (8) has the simple solution

$$y_0(z) = a_0(t) + a_1(t)z + a_2(t)z^2 + a_3(t)z^3 + a_4(t)z^4 + a_5(t)z^5. \qquad (9)$$

Factors $a_k, k = \overline{1,5}$ are from boundary condition and the equation (9)

$$\begin{aligned} a_0(t) &= 0, & a_3(t) &= -\tfrac{1}{6}\tfrac{m_z l^3}{EJ}((a + \tfrac{1}{2}l)\ddot{\alpha} - \ddot{y}_c), \\ a_1(t) &= 0, & a_4(t) &= \tfrac{1}{24}\tfrac{m_z l^3}{EJ}(a\ddot{\alpha} - \ddot{y}_c), \\ a_2(t) &= \tfrac{1}{6}\tfrac{m_z l^3}{EJ}((\tfrac{3}{2}a + l)\ddot{\alpha} - \tfrac{3}{2}\ddot{y}_c), & a_5(t) &= \tfrac{1}{120}\tfrac{m_z l^3}{EJ}(l\ddot{\alpha}). \end{aligned} \qquad (10)$$

6. Oscillating component of the solution

The equation describing elasticity of the rod for oscillating component $\Delta y(z,t)$ of the solution (7) look like

$$\frac{EJ}{m_z l^3}(1+h\frac{\partial}{\partial t})\frac{\partial^4 \Delta y(z,t)}{\partial z^4}+\frac{\partial^2 \Delta y(z,t)}{\partial t^2}=-\frac{\partial^2 y_0(z,t)}{\partial t^2} \qquad (11)$$

with zero boundary conditions.
The solution of equation (11) look like

$$\Delta y(z,t)=\sum_{k=0}^{\infty} B_k(t)A_k(z). \qquad (12)$$

Substitute the solution (12) in the original equation (11), consider

$$A_k^{''''}(z)=\mu_k^4 A_k(z), \qquad (13)$$

and exclude the higher-order derivatives $\dddot{\alpha}$ and $\ddddot{y_c}$. Then the equation describing elasticity of the rod for oscillating component $\Delta y(z,t)$ will become

$$\ddot{B}_k(t)=-\frac{1}{\varepsilon}\mu_k^4(B_k(t)+h\dot{B}_k(t))+\frac{1}{6}\frac{\varepsilon}{w}(Q_4\ddot{u}+Q_5 u+Q_6\alpha+\sum_{i=1}^{\infty}Q_7 B_i).$$

7. The system of equations of oscillation of a satellite

The equations of the original system (1) look like

$$\begin{cases} \ddot{y}_c = Q_0 u + Q_1 \alpha + \sum\limits_{i=1}^{\infty} P_i B_i \\ \ddot{\alpha} = Q_2 u + Q_3 \alpha + \sum\limits_{i=1}^{\infty} S_i B_i \\ \ddot{B}_k(t) = -\frac{1}{\varepsilon} \mu_k^4 (B_k(t) + h\dot{B}_k(t)) + \frac{1}{6} \frac{\varepsilon}{w} (Q_4 \ddot{u} + Q_5 u + Q_6 \alpha + \sum\limits_{i=1}^{\infty} Q_7 B_i). \end{cases}$$

Control look like $u = -\phi_1 \alpha - \phi_2 \dot{\alpha}$.
The system of equations of oscillations of a satellite will become

$$\begin{cases} \ddot{\alpha} = Q_8 \alpha + Q_9 \dot{\alpha} + \sum\limits_{i=1}^{N} S_i B_i \\ \ddot{B}_k(t) = Q_{10}(B_k(t) + h\dot{B}_k(t)) + Q_{11}\alpha + Q_{12}\dot{\alpha} + \sum\limits_{i=1}^{N} \Lambda_{ki} B_k + \sum\limits_{i=1}^{N} R_{ki} \dot{B}_i. \end{cases}$$

This system of equations of order $(2 + 2N)$, here N – number of taking into account modes.

8. The system of equations of oscillation of a satellite

The system of equations of oscillations of a satellite

$$\begin{cases} \ddot{\alpha} = Q_8\alpha + Q_9\dot{\alpha} + \sum\limits_{k=1}^{N} S_k B_k \\ \ddot{B}_k(t) = Q_{10}(B_k(t) + h\dot{B}_k(t)) + Q_{11}\alpha + Q_{12}\dot{\alpha} + \sum\limits_{i=1}^{N} \Lambda_{ki} B_k + \sum\limits_{i=1}^{N} R_{ki}\dot{B}_i. \end{cases}$$

The state vertor

$$\begin{pmatrix} \alpha & \dot{\alpha} & B_1 & B_2 & \dots & B_N & \dot{B}_1 & \dot{B}_2 & \dots & \dot{B}_N \end{pmatrix}^T.$$

Matrix of the system

$$\begin{pmatrix}
0 & 1 & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\
Q_8 & Q_9 & S_1 & S_2 & \dots & S_N & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 1 & \dots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & \dots & 1 \\
Q_{11}^1 & Q_{12}^1 & Q_{10}^1 + \Lambda_{11} & \Lambda_{12} & \dots & \Lambda_{1N} & hQ_{10}^1 + R_{11} & \dots & R_{1N} \\
Q_{11}^2 & Q_{12}^2 & \Lambda_{21} & Q_{10}^2 + \Lambda_{22} & \dots & \Lambda_{2N} & R_{21} & \dots & R_{2N} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
Q_{11}^N & Q_{12}^N & \Lambda_{N1} & \Lambda_{N2} & \dots & Q_{10}^N + \Lambda_{NN} & R_{N1} & \dots & hQ_{10}^N + R_{NN}
\end{pmatrix}$$

9. Modelling. The stability diagram

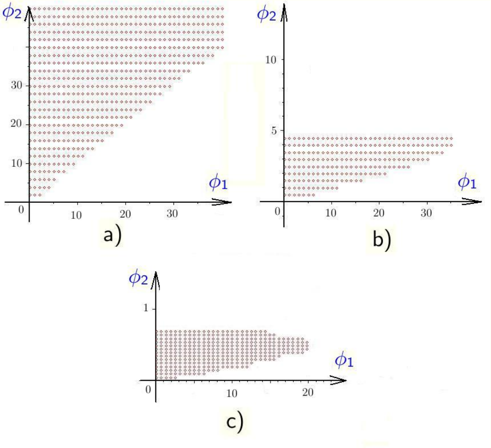

Figure : Stability diagram: a) 1 mode; b) 2 modes; c) 3 modes.

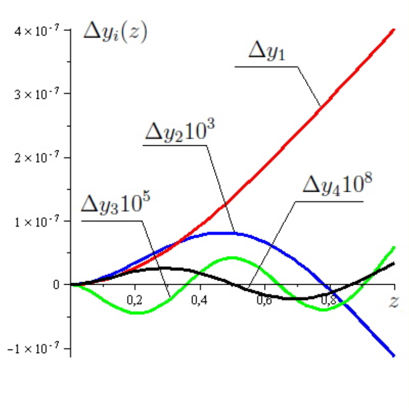

Figure : Graph mode contribution multiplied by the corresponding coefficients

11. Modelling. Graph of function $\alpha(t)$

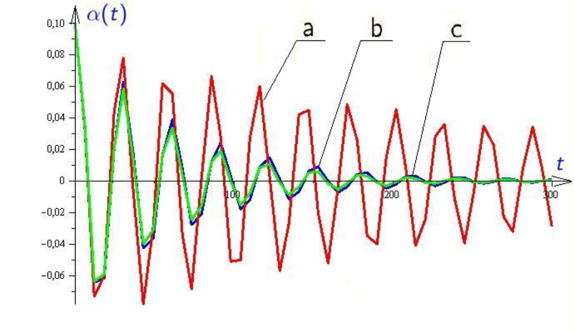

Figure : The graph of $\alpha(t)$: a) 1 mode; b) 2 modes; c) 3 modes.

12. The results

1. Using the principle of separation of movement on the slow quasi-static and fast oscillating we obtain a system of ordinary differential equations, describing the controlled movement of the satellite with an elastic rod that allows to take into account any number of vibrational modes;
2. It is established, that the simple linear regulator, used for the stabilization of orientation of the rigid satellite, ensures the stability of the orientation of the satellite with flexible element in the proper choice of the coefficients of the regulator;