Numerical identification of 3-rd order object

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Subject of system identification

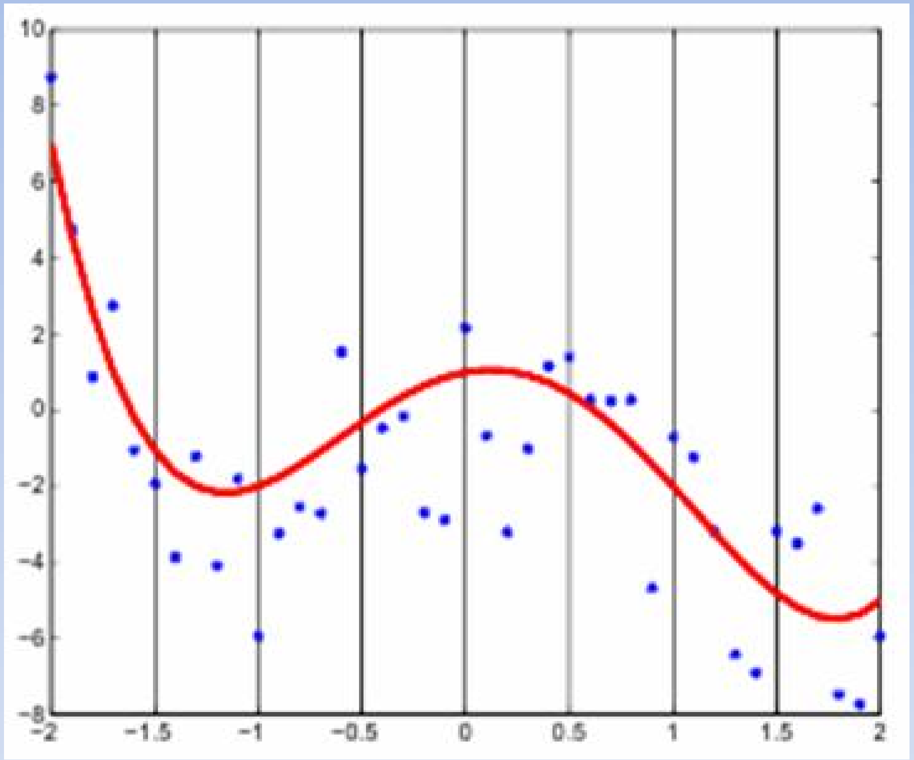

Fig.1. Plot of x-values

G(x) is an unknown function that obtained from sequence of x-values. The problem is to construct an estimate Gp(x) which will be the best approximation of G(x).

Subject of system identification

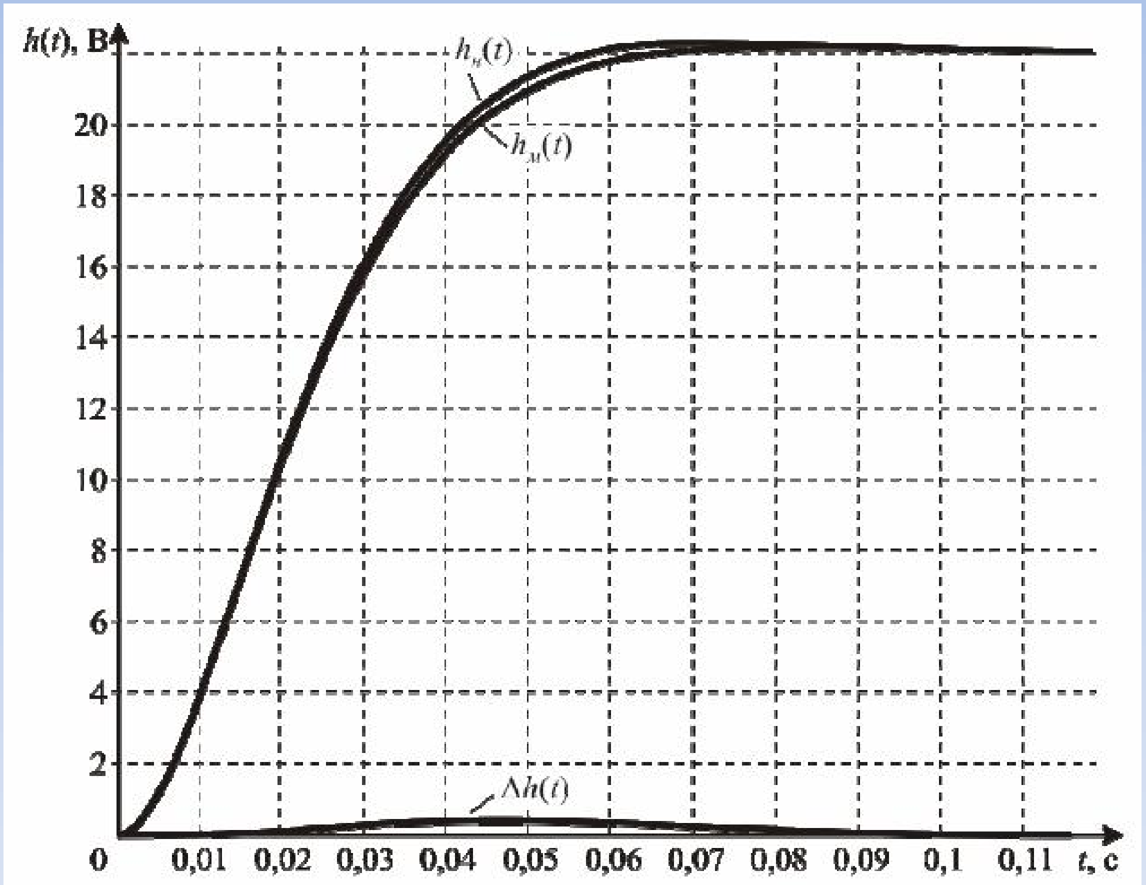

Fig.2. Plots of step responses

Foundations of the method

Real integral transform

$$F(\delta) = \int_0^\infty f(t) e^{-\delta t} dt, \delta \in [C_\nu, \infty), C_\nu \geq 1$$

Properties of the transform:

1) Images $F(\delta)$ depend of real argument δ ;
2) Images are obtained from Laplace transform $F(p)$ with help of

$$p \to \delta$$

3) Continuous function can be presented as a discrete function $F(\delta_i), i = 1, 2, .., \eta$ and presented in system of linear equations if we need.

Foundations of the method

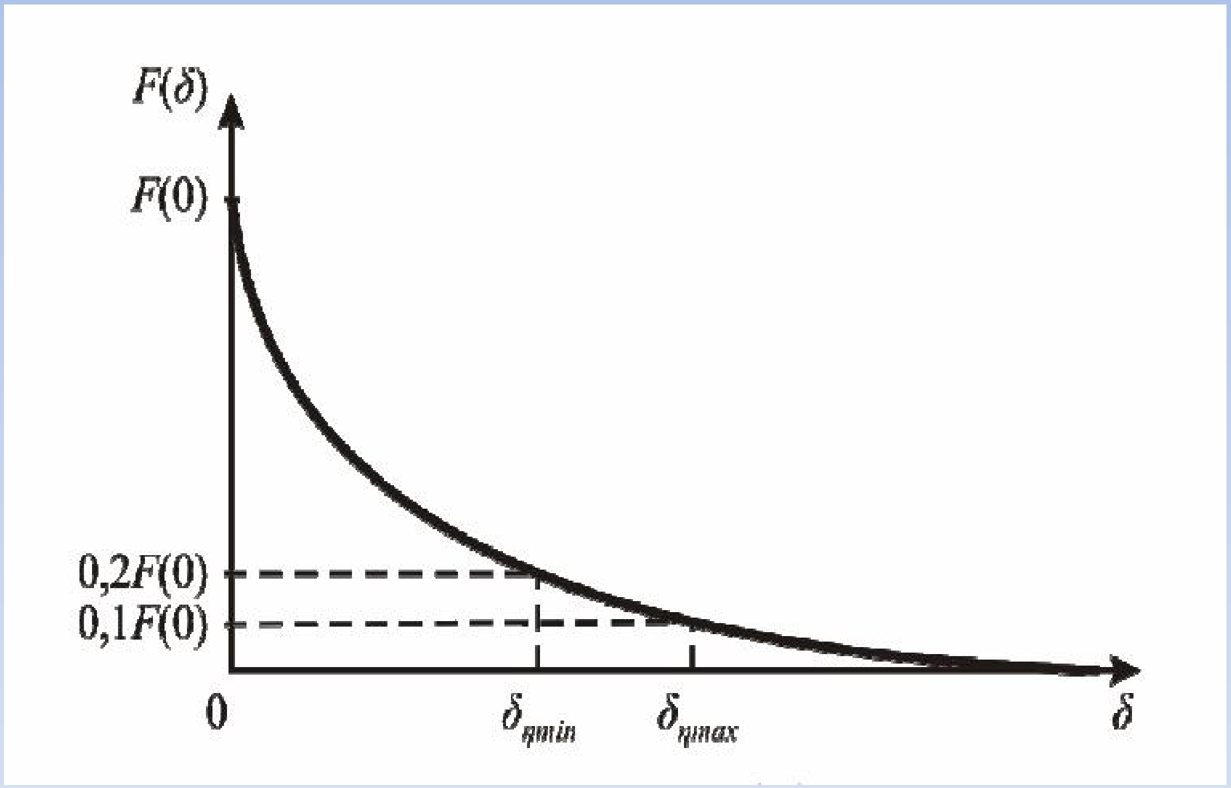

Fig.3. Plot of $F(\delta)$ if m<n

Where interpolation points choose from form of normal distribution:

$$\delta_i = \delta_1 + \frac{(\delta_\eta - \delta_1)(i-1)}{\eta - 1}.$$

$$\delta_1 = \frac{-\ln(0.01)}{t_{\text{п.п.}}}.$$

Last point chooses from relation:

$$F(\delta_\eta) = (0.1 \div 0.2)F(\delta_1).$$

Application to the parameter identification

Given step response in analytical form:

$$h_3(t) = Ae^{(-0.125-0.992i)t} + Be^{(-0.125+0.992i)t} - 1.35e^{-3t} + 25,$$

where $A = \frac{(-49.6i+293.75)}{(10.6i-21.6)}, B = \frac{(49.6i+293.75)}{(-10.6i-21.6)}.$

Given structure of transfer function:

$$W(p) = \frac{k}{a_2 p^2 + a_1 p + 1}.$$

The problem is to find the unknown coefficients of W(p) such that step response of obtained function will be similar to given response with same value of error function:

$$\varepsilon = \sum_{i=1}^{n} \left| h_p(t_i) - h(t_i) \right|$$

Application to the parameter identification

Identification algorithm:

1. Given m values of step response $h(t_j)$.
2. Estimating a real transfer function of object $W(\delta) = \delta \sum_{j=1}^{m} h(t_j)\, e^{\delta t_j} \Delta t_j$.
3. Choosing a first and last points of interpolation from next conditions:

$$\delta_1 = \frac{-\ln(0.01)}{t_p};\ W(\delta_n) = (0.1 \div 0.2) \cdot W(\delta_1).$$

4. Solving the system of linear equations: $W(\delta_i) = \frac{b_m \delta_i^{\,m} + b_{m-1} \delta_i^{\,m-1} + \dots + b_1 \delta_i + b_0}{a_n \delta_i^{\,n} + a_{n-1} \delta_i^{\,n-1} + \dots + a_1 \delta_i + a_0}$,

where $i = 1..m + n + 2$.

5. We obtain the parameters ($b_i, a_j; i = 1..m, j = 1..n$) and calculate an error:

$$\varepsilon = \sum_{i=1}^{n} \left| h_p(t_i) - h(t_i) \right|$$

Results

$h_{\text{п}}(t)$ -is given function.

$h_{\text{э}}(t)$ - is obtained function.

$\varepsilon(t)$ -is error function.

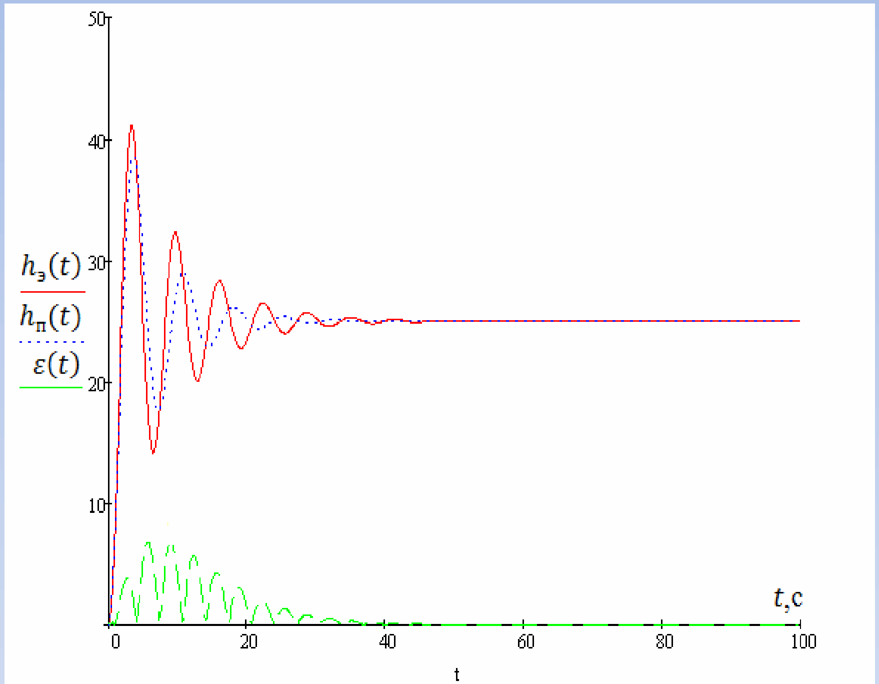

Fig.4. Plots of step responses

Obtained transfer function of object:

$$W(p) = \frac{25}{1.265p^2 + 0.438p + 1}.$$

THANK YOU FOR YOUR ATTENTION!