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Laboratory №1

"Dynamic control systems"

Discrete observer based compensator

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An arbitrary dynamic compensator

There are a given plant

$$\mathbf{x}_{k+1} = A\mathbf{x}_k + B\mathbf{u}_k, \tag{1}$$

$$\mathbf{y}_k = C\mathbf{x}_k + D\mathbf{u}_k, \tag{2}$$

and an arbitrary dynamic compensator

$$\mathbf{x}^R_{k+1} = A^R\mathbf{x}^R_k + B^R\mathbf{y}_k, \tag{3}$$

$$\mathbf{u}_k = C^R\mathbf{x}^R_k + D^R\mathbf{y}_k. \tag{4}$$

The arbitrary dynamic compensator

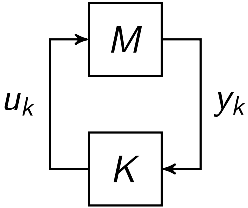

Рис.1. The closed–loop system

Let us interconnect (1)–(4)

The compensator dynamics

$$x_{k+1}^R = \left\{ A^R + B^R D \left(I - D^R D \right)^{-1} C^R \right\} x_k^R + \\ + B^R D \left(I - D^R D \right)^{-1} D^R C x_k, \tag{5}$$

$$u_k = \left(I - D^R D \right)^{-1} C^R x_k^R + \left(I - D^R D \right)^{-1} D^R C x_k, \tag{6}$$

The Luenberger observer

The Luenberger observer can be built if there exist $T \in \mathbb{R}^{n_R \times n}$, $F \in \mathbb{R}^{n_R \times n_R}$, $G \in \mathbb{R}^{n_R \times p}$ and $B_1 \in \mathbb{R}^{n_R \times m}$, such that

$$\hat{z}_{k+1} = F\hat{z}_k + Gy_k + B_1 u_k \tag{7}$$

holds $z_k - \hat{z}_k \xrightarrow[k\to\infty]{} 0$, где $\hat{z}_k = T\hat{x}_k$. Let us apply (2) by (7)

$$\hat{z}_{k+1} = F\hat{z}_k + GCx_k + (GD + B_1)\, u_k. \tag{8}$$

By definition, put $GD + B_1 = TB$. The observre dynamics is described by the equation

$$\hat{z}_{k+1} = F\hat{z}_k + GCx_k + TBu_k. \tag{9}$$

The Luenberger observer

The error dynamics equation is

$$z_{k+1} - \hat{z}_{k+1} = TAx_k - F\hat{z}_k - GCx_k,$$

where $z_k = Tx_k$ is given from (1). If $z_k - \hat{z}_k \xrightarrow[k\to\infty]{} 0$ holds true, one can get

The Luenberger equation

$$TA - FT - GC = 0. \tag{10}$$

Let us apply (6) to (9)

The observer dynamics

$$\hat{z}_{k+1} = \left\{ F + TB\left(I - D^R D\right)^{-1} C^R \right\} \hat{z}_k + \\ + \left\{ G + TB\left(I - D^R D\right)^{-1} D^R \right\} C x_k. \quad (11)$$

Let us compare (5) and (11)

$$A^R + B^R D\left(I - D^R D\right)^{-1} C^R = F + TB\left(I - D^R D\right)^{-1} C^R,$$

$$B^R\left(I + D\left(I - D^R D\right)^{-1} D^R\right) = G + TB\left(I - D^R D\right)^{-1} D^R.$$

The transformation matrix

The unknown transformation matrix T is a solution of a non–cymmetric Riccati equation

The Riccati equation

$$T(A+B\widetilde{D}D^RC)-(A^R+B^RD\widetilde{D}C^R)T+ \\ +TB\widetilde{D}C^RT-B^R\left(I+D\widetilde{D}D^R\right)C=0. \qquad (12)$$

The unknown matrices F and G are computed as solutions of the algebraic equations

$$F=A^R+B^RD\widetilde{D}C^R-TB\widetilde{D}C^R, \qquad (13)$$

$$G=B^R\left(I+D\widetilde{D}D^R\right)-TB\widetilde{D}D^R. \qquad (14)$$

The augmented–order compensator

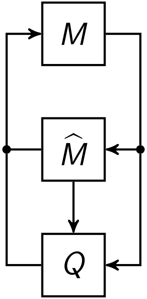

Рис.2.The compensator's decomposition

The goal is to design an observer from the arbitrary dynamic compensator.

The augmented–order compensator

The statement of a problem is to find some static gains and dynamic Youla parameter such that observer–based compensator structure in figure (2) is equal to the original compensator.

$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k + K_f\left(y_k - C\hat{x}_k - Du_k\right), \quad (15)$$

$$x^Q_{k+1} = A^Q x^Q_k + B^Q\left(y_k - C\hat{x}_k - Du_k\right), \quad (16)$$

$$u_k = -K_c\hat{x}_k + C^Q x^Q_k + D^Q\left(y_k - C\hat{x}_k\right). \quad (17)$$

The closed–loop matrix A_{cl}

$$A_{cl} = \begin{bmatrix} A - B\check{D}K_c & B\check{D}C^Q & B\check{D}\left(K_c + D^Q C\right) \\ 0 & A^Q & B^Q C \\ 0 & 0 & A - K_f C \end{bmatrix},$$

where $\check{D} = (I - D)^{-1}$.

Consider a Schur decomposition of the matrix F

$$F = V\tilde{F}V^* = [V_1 \;\; V_2]\left[\begin{array}{cc} \tilde{F}_{11} & \tilde{F}_{12} \\ 0 & \tilde{F}_{22} \end{array}\right]\left[\begin{array}{c} V_1^* \\ V_2^* \end{array}\right], \tag{18}$$

where $VV^* = I_{n_R}$, $\tilde{F}_{11} \in \mathbb{R}^{n_R - n \times n_R - n}$ и $\tilde{F}_{22} \in \mathbb{R}^{n \times n}$. Let us inroduce a new variable

$$\hat{z}_k = [V_1 \;\; V_2]\left[\begin{array}{c} \omega_k^1 \\ \omega_k^2 \end{array}\right]. \tag{19}$$

$$\left[\begin{array}{c} \omega_{k+1}^1 \\ \omega_{k+1}^2 \end{array}\right] = \left[\begin{array}{cc} \tilde{F}_{11} & \tilde{F}_{12} \\ 0 & \tilde{F}_{22} \end{array}\right]\left[\begin{array}{c} \omega_k^1 \\ \omega_k^2 \end{array}\right] + \left[\begin{array}{c} V_1^* \\ V_2^* \end{array}\right] G y_k + \left[\begin{array}{c} V_1^* \\ V_2^* \end{array}\right] B_1 u_k. \tag{20}$$

By definition, put

$$\begin{bmatrix} V_1^* \\ V_2^* \end{bmatrix} G = \begin{bmatrix} \tilde{G}_1 \\ \tilde{G}_2 \end{bmatrix}, \begin{bmatrix} V_1^* \\ V_2^* \end{bmatrix} T = \begin{bmatrix} \tilde{T}_1 \\ \tilde{T}_2 \end{bmatrix} \text{and} \begin{bmatrix} V_1^* \\ V_2^* \end{bmatrix} B = \begin{bmatrix} \tilde{B}_1 \\ \tilde{B}_2 \end{bmatrix}. \tag{21}$$

Then the Luenberger equation (10) is splited on two

$$\tilde{T}_1 A - \tilde{F}_{11}\tilde{T}_1 - \tilde{F}_{12}\tilde{T}_2 - \tilde{G}_1 C = 0, \tag{22}$$

$$\tilde{T}_2 A - \tilde{F}_{22}\tilde{T}_2 - \tilde{G}_2 C = 0. \tag{23}$$

The unknown matrices $\tilde{F}_{22}$ and $\tilde{F}_{12}$ are

$$\tilde{F}_{22} = \left(\tilde{T}_2 A - \tilde{G}_2 C\right) \tilde{T}_2^{-1}, \tag{24}$$

$$\tilde{F}_{12} = \left(\tilde{T}_1 A - \tilde{F}_{11}\tilde{T}_1 - \tilde{G}_1 C\right) \tilde{T}_2^{-1}. \tag{25}$$

Let us change the variables

$$\hat{x}_k = \tilde{T}_2^{-1}\omega_k^2, \tag{26}$$
$$x_k^Q = \omega_k^1 - \tilde{T}_1\hat{x}_k, \tag{27}$$

then the equations (15) and (16) change to

$$\hat{x}_{k+1} = A\hat{x}_k + \tilde{T}_2^{-1}\left(\tilde{G}_2 D + B_2\right) u_k, \tag{28}$$
$$x_{k+1}^Q = \tilde{F}_{11}x_k^Q + \left[\tilde{G}_1 D + \tilde{B}_1 - \tilde{T}_1\tilde{T}_2^{-1}\left(\tilde{G}_2 D + \tilde{B}_2\right)\right] u_k. \tag{29}$$

The state of the compensator includes both states $\hat{x}_k$ and x_k^Q

$$x_k^R = [V_1\,T]\begin{bmatrix} x_k^Q \\ \hat{x}_k \end{bmatrix}.$$

Then control law is

$$u_k = C^R x_k^R + D^R y_k = C^R T\hat{x}_k + C^R V_1 x_k^Q + D^R y_k =$$
$$= \left(C^R T + D^R C\right)\hat{x}_k + C^R V_1 x_k^Q + D^R\left(y_k - C\hat{x}_k\right). \tag{30}$$

The calculations of the model parameters

All sougth for matrices of 2 are calculated according equations

$$K_f = \left[B - (V_2^* T)^{-1} V_2^* (GD + B_1)\right] D^{-1}, \tag{31}$$

$$K_c = -\left(C^R T + D^R C\right), \tag{32}$$

$$A^Q = \tilde{F}_{11} = V_1^* F V_1, \tag{33}$$

$$B^Q = V_1^* \left(I - T (V_2^* T)^{-1} V_2^*\right) (GD + B_1), \tag{34}$$

$$C^Q = C^R V_1, \tag{35}$$

$$D^Q = D^R, \tag{36}$$

where F, G, V_1^* и V_2^* — are special matrices.

The observer–based compensator

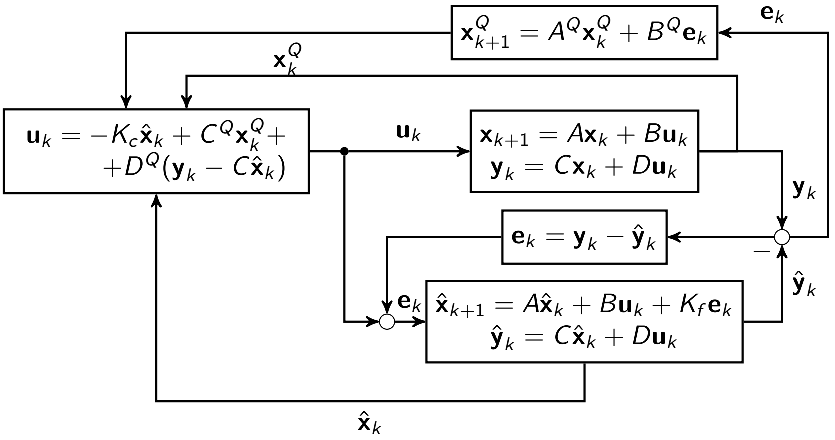

Рис.3.The closed–loop system

The rediced–order compensator

Let us consider $n_R < n$. Then the compensator dynamics is

$$\hat{z}_{k+1} = F\hat{z}_k + Gy_k + B_1 u_k, \tag{37}$$

$$\hat{x}_k = H_1\hat{z}_k + H_2 y_k, \tag{38}$$

$$u_k = -K_c\hat{x}_k + D^Q\left(y_k - C\hat{x}_k\right). \tag{39}$$

The parameters of the model are calculated according to

$$C^R = -\left(K_c + D^Q C\right) H_1, \tag{40}$$

$$D^R = D^Q - K_c H_2 - D^Q C H_2, \tag{41}$$

$$H_1 T + H_2 C = I. \tag{42}$$

$$K_c = -C^R T - D^R C. \tag{43}$$

The rediced–order compensator

If $n^R > n - p$ holds true, then

$$H_1 = \left[T^T T + C^T C\right]^{-1} T^T,$$
$$H_2 = \left[T^T T + C^T C\right]^{-1} C^T.$$

The matrix D^Q is calculated by means of (40) and (41).
If $n^R < n - p$ there is no proper techniques of the reduced–order observer design.

If $n^R = n$, then the transformation matrix T of the state space is square. The Schur decomposition (18) is singular. It means that the matrix V_2 has full rank, furthermore the matrix V_1 is empty. Let us remark that the Youla parameter reduces to the static gain $D^Q = D^R$.

$$K_f = \left[B - T^{-1}\left(GD + B_1\right)\right] D^{-1},$$
$$K_c = -\left(C^R T + D^R C\right),$$
$$D^Q = D^R.$$

Future aims

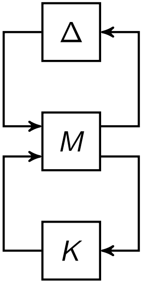

Рис.4.The uncertanity contain system

- introducing of an uncertanities (nonstructured and structured) to the model;
- design a compensator in the case of disturbance.

THANKS FOR ATTANTION!

Решение несимметричного уравнения Риккати

1) поиск n–мерного инвариантного подпространства S матрицы A_{cl}. В нашем случае матрица замкнутой системы имеет вид

$$A_{cl} = \begin{bmatrix} A + B\widetilde{D}D^R C & B\widetilde{D}C^R \\ B^R\left(I + D\widetilde{D}D^R\right)C & A^R + B^R D\widetilde{D}C^R \end{bmatrix}.$$

Инвариантное подпространство вычисляется в виде решения следующего уравнения

$$A_{cl}U = U\Lambda; \qquad (44)$$

2) матрица U имеет блочную структуру

$$U = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}, \ U_1 \in \mathbb{R}^{n\times n};$$

3) решение уравнения (12) определяется как $T = U_2^{-1} \cdot U_1$.