# Power Allocation in OFDMA Networks: An ADMM Approach

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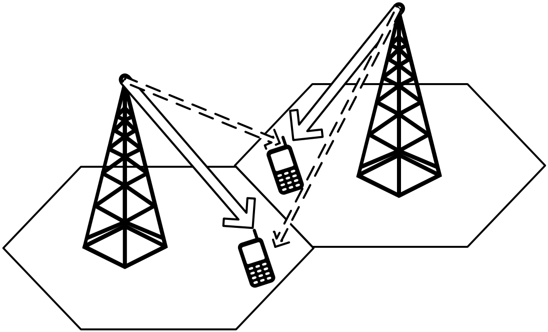
# Introduction

Considered OFDMA multicell wireless network

- Objective: increase throughput
- Obstacle: inter-cell interference
- One of possible solutions: dynamic resource management (scheduling and **power allocation**)



## Problem Statement

Weighted sum rate maximization problem (**nonconvex**) [NCVX]:

$$\max_{\mathbf{0}\leq \boldsymbol{P} \leq \boldsymbol{S}_{max}} \sum_{l=1}^{L}\sum_{n=1}^{N} w_{k(l,n)} \log(1+\mathrm{SINR}^{n}_{lk(l,n)}(\boldsymbol{P}^n))$$
$$\text{s.t.} \sum_{n=1}^{N} \boldsymbol{P}^n \leq \boldsymbol{P}_{max},$$

$L$ – number of cells which share frequency resource divided by $N$ blocks/frequency tones;
$k(l,n)$ – a user $k$ is assigned to the $n$th frequency tone in the $l$th cell;
$w_{k(l,n)} > 0$ – users weights;
$\boldsymbol{P}_{max} \in \mathbb{R}^L$ and $\boldsymbol{S}_{max} \in \mathbb{R}^{NL}$

$\boldsymbol{P}^n = (P_1^n, \ldots, P_L^n)^T \in \mathbb{R}^L$, $P_l^n$ – DL transmit power spectral density;
Signal-to-interference-plus-noise ratio (SINR):

$$\mathrm{SINR}_{lk(l,n)}^n(\boldsymbol{P}^n) = \frac{P_l^n G_{lk(l,n)}^n}{\sigma^2 + \sum\limits_{j=1, j \neq l}^{L} P_j^n G_{jk(l,n)}^n}$$

$G_{jk(l,n)}^n = |h_{jk(l,n)}^n|^2$, $h_{jk(l,n)}^n$ is the channel response between base station $j$ and user $k$ in cell $l$ in $n$ frequency tone; $\sigma^2$ – noise;

## Popular approaches to solve the considered nonconvex problem

1. dual decomposition
2. convex relaxation and dual decomposition

## Pros and cons of dual decomposition

+ one problem is decomposed into $N$ smaller problem
- isn't as robust as augmented Lagrangian methods

## The two main ingredients of the proposed method are

1. successive convex relaxations based on SCALE approach
2. Alternating Direction Method of Multipliers (ADMM) for solving obtained convex problems

+ ADMM has both separability and robustness

# Convex relaxation based on SCALE approach

$$\alpha \log z + \beta \leq \log(1+z),\ \alpha = \frac{z_0}{1+z_0},\ \beta = \log(1+z_0) - \frac{z_0}{1+z_0}\log z_0$$

Applying this inequality and transformation $\widetilde{P}_l^n = \log(P_l^n)$ we get **convex** problem [SCALE]:

$$\max_{\widetilde{\boldsymbol{P}} \leq \log(\boldsymbol{S}_{max})} \sum_{n=1}^{N} \phi_n(\widetilde{\boldsymbol{P}}^n) = \sum_{n=1}^{N}\sum_{l=1}^{L} w_{k(l,n)}\alpha_l^n \log(\mathrm{SINR}_{lk(l,n)}^n(e^{\widetilde{\boldsymbol{P}}^n}))$$

$$\text{s.t.} \sum_{n=1}^{N} \exp(\widetilde{\boldsymbol{P}}^n) \leq \boldsymbol{P}_{max}$$

Iteratively improve a lower-bound for nonconvex problem by successively solving relaxed (convex) problem with $\alpha$ calculated at $z_0$ equal to SINR from previous iteration.

## Dual problem

$$\max_{\boldsymbol{\lambda}\in\mathbb{R}^L_+} g(\boldsymbol{\lambda}) = \sum_{n=1}^{N} g_n(\boldsymbol{\lambda}),$$

$$g_n(\boldsymbol{\lambda}) = \inf_{\widetilde{\boldsymbol{P}}^n \le \log(\boldsymbol{S}^n_{max})} -\phi_n(\widetilde{\boldsymbol{P}}^n) + \left\langle \boldsymbol{\lambda}, e^{\widetilde{\boldsymbol{P}}^n} - \frac{\boldsymbol{P}_{max}}{N} \right\rangle,$$

where $\boldsymbol{\lambda}$ – Lagrange multipliers, or equivalently

$$\min_{\boldsymbol{\lambda}\in\mathbb{R}^L,\ \boldsymbol{z}\in\mathbb{R}^{NL}_+} \sum_{n=1}^{N} -g_n(\boldsymbol{z}_n)$$

$$\text{s.t.} \boldsymbol{\lambda} - \boldsymbol{z}_n = 0,\ n = 1, \dots N,$$

$\boldsymbol{z}$ – artificial variables.

Augmented Lagrangian for above problem:

$$L_\rho(\boldsymbol{\lambda}, \boldsymbol{z}, \boldsymbol{y}) = \sum_{n=1}^{N} \left[ -g_n(\boldsymbol{z}_n) + \langle \boldsymbol{y}_n, \boldsymbol{\lambda} - \boldsymbol{z}_n \rangle + \frac{\rho}{2} ||\boldsymbol{\lambda} - \boldsymbol{z}_n||^2 \right],$$

$\boldsymbol{y}$ – Lagrange multipliers; $\rho > 0$ – penalty parameter.

**ADMM applied to dual problem:**

$$\boldsymbol{\lambda}^{(k+1)} = \arg\min_{\boldsymbol{\lambda} \in \mathbb{R}^L} \left\langle \sum_{n=1}^{N} \boldsymbol{y}_n^{(k)}, \boldsymbol{\lambda} \right\rangle + \frac{\rho}{2} \sum_{n=1}^{N} \left|\left| \boldsymbol{\lambda} - \boldsymbol{z}_n^{(k)} \right|\right|^2,$$

$$\boldsymbol{z}_n^{(k+1)} = \arg\max_{\boldsymbol{z}_n \geq 0} g_n(\boldsymbol{z}_n) + \left\langle \boldsymbol{y}_n^{(k)}, \boldsymbol{z}_n \right\rangle - \frac{\rho}{2} \left|\left| \boldsymbol{\lambda}^{(k+1)} - \boldsymbol{z}_n \right|\right|^2,$$

$$\boldsymbol{y}_n^{(k+1)} = \boldsymbol{y}_n^{(k)} + \rho \left( \boldsymbol{\lambda}^{(k+1)} - \boldsymbol{z}_n^{(k+1)} \right), \; n = 1, \ldots N.$$

For $\lambda$ and $\boldsymbol{z}_n$ we have a solution in an explicit form.

Final algorithm:

Choose $\rho > 0$, $\boldsymbol{z}_n^{(0)}$, $\boldsymbol{y}_n^{(0)}$, $n = 1, \ldots, N$; then repeat

1. $\boldsymbol{\lambda}^{(k+1)} = \frac{1}{N} \sum_{n=1}^{N} \left( \boldsymbol{z}_n^{(k)} - \frac{1}{\rho} \boldsymbol{y}_n^{(k)} \right)$
2. For each $n = 1, \ldots, N$ solve in parallel $N$ convex problems

$$(\widetilde{\boldsymbol{P}}^n)^{(k+1)} = \underset{\widetilde{\boldsymbol{P}}^n \leq \log(\boldsymbol{S}^n_{max})}{\arg\min} \quad -\phi_n(\widetilde{\boldsymbol{P}}^n) + \frac{\rho}{2} \sum_{l=1}^{L} \left( \left[ \lambda_l^{(k+1)} + \frac{1}{\rho} \left( y_{ln}^{(k)} + e^{\widetilde{P}_l^n} - \frac{P_{l,max}}{N} \right) \right]^+ \right)^2$$

$$z_{ln}^{(k+1)} = \left[ \lambda_l^{(k+1)} + \frac{1}{\rho} \left( y_{ln}^{(k)} + e^{(\widetilde{P}_l^n)^{(k+1)}} - \frac{P_{l,max}}{N} \right) \right]^+ ,$$

3. $\boldsymbol{y}_n^{(k+1)} = \boldsymbol{y}_n^{(k)} + \rho \left( \boldsymbol{\lambda}^{(k+1)} - \boldsymbol{z}_n^{(k+1)} \right), \; n = 1, \ldots, N,$

where we use the notation $x^+ = max(0, x)$.

## Numerical examples. Scenario:

Network with wrap-around topology
Number of cells: $L = 37$
Inter cell distance: 2500 m
System frequency: 2110 MHz
System bandwidth: 10 MHz
Number of resource blocks: $N = 50$
Maximum cell transmit power $P_{l,max} = 40$ W
Users: 20 users at each cell distributed close to the cell-edge at the distance 0.6..0.9 of cell radius and moving with the speed of 3 km/h
Standard deviation for the log-normal fading: 8 dB
Log-normal correlation downlink: 0.5
Log-normal fading correlation distance: 100 m
Channel model: ITU PedA
Antenna Techniques: SISO
Thermal Noise: 5.7e-15 W
Traffic Model: Full-buffer

# Results

ADMM was compared with subgradient (SG) and ellipsoid (EL) methods which are used for solving dual problems (NCVX and SCALE)

**[NCVX]:**

| | | |
|---|---|---|
| SG | OFV | 6614.9 |
| | NII | 228 |

**[SCALE]:**

| NOI | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SG | OFV | 6619.9 | 6622.5 | 6623.1 | 6623.1 | 6623.2 | 6622.1 | 6622.8 | 6623.2 | 6622.1 | 6622.9 |
| | NII | 68 | 30 | 193 | 9 | 20 | 27 | 24 | 5 | 42 | 34 |
| ADMM | OFV | 6621 | 6623.1 | 6623.2 | 6623.2 | 6623.2 | 6623.2 | 6623.2 | | | |
| | NII | 2 | 1 | 1 | 1 | 1 | 1 | 1 | | | |
| EL | OFV | 6619.4 | 6621.5 | 6621.7 | 6621.7 | 6621.7 | 6621.7 | 6621.7 | | | |
| | NII | 160 | 164 | 170 | 170 | 169 | 170 | 170 | | | |

NOI – number of outer iterations ($\alpha-$update)
NII – number of inner iterations (Lagrange multipliers update)
OFV – objective function value (weighted sum rate)

# Conclusions

- Developed method based on SCALE+ADMM approach for solving nonconvex weighted sum rate maximization problem
- Numerical simulations demonstrate the performance (less number of Lagrange multipliers updates) of proposed method compared with the existing ones: NCVX+SG (W. Yu et al) and SCALE+EL (L. Venturino et al)

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Thank you!

## Stopping criteria (Boyd et al):

$$\left|\left|\boldsymbol{r_{pri}}^{(k)}\right|\right|_2 \leq \epsilon_{pri}$$

and

$$\left|\left|\boldsymbol{r_{dual}}^{(k)}\right|\right|_2 \leq \epsilon_{dual}$$

where the primal and dual residuals

$$\boldsymbol{r_{pri}}^{(k)} = \left[\boldsymbol{\lambda}^{(k)} - \boldsymbol{z_n}^{(k)}\right]_1^N$$

$$\boldsymbol{r_{dual}}^{(k)} = -\rho\left(\boldsymbol{z}^{(k)} - \boldsymbol{z}^{(k-1)}\right)$$