Compensation of Multi-Sinusoidal Disturbance for Linear Plant with Input Delay

Oleg I. Borisov
Research Supervisor: Anton A. Pyrkin

ITMO University

Traditional School for Young Scientists VI
"Control, Information, Optimization"

Grigorchikovo, Russia
June 22-29, 2014

Outline

Plant Model

Consider the linear plant

$$\dot{x}(t) = Ax(t) + Bu(t-D) + B\delta(t), \quad (1)$$
$$y(t) = Cx(t) + \alpha\delta(t), \quad (2)$$

where $x \in \mathbb{R}^n$ is unknown, $y(t)$ is measurable, $u(t)$ has initial condition $u(t-D) = 0$ for $t < D$, $D \geq 0$ is known constant delay, A, B, C are corresponding matrices, $\alpha \neq 0$.
The input disturbance $\delta(t)$ has a view

$$\delta(t) = \sigma + \sum_{i=1}^{k} \left[\mu_i \sin(\omega_i t) + \nu_i \cos(\omega_i t)\right]. \quad (3)$$

Purpose of Control

The objective is to find the control $u(t)$ that achieves regulation of the state variables

$$\lim_{t\to\infty} x(t) = 0\,, \quad (4)$$

under the following assumptions
Assumption 1. *A, B, C, and α are known.*
Assumption 2. *(A, B, C) is a completely controllable and observable triple.*
Assumption 3. *A lower bound on the disturbance frequencies is known.*

Frequency Estimation

Consider the linear observer

$$\dot{\hat{x}}_1(t) = A\hat{x}_1(t) + Bu(t-D) + L\tilde{y}_1(t), \tag{5}$$

$$\hat{y}_1(t) = C\hat{x}_1(t). \tag{6}$$

Then the observer error is given by

$$\tilde{x}_1(t) = x(t) - \hat{x}_1(t), \tag{7}$$

$$\dot{\tilde{x}}_1(t) = (A - LC)\tilde{x}_1(t) + B\delta(t) - L\alpha\delta(t), \tag{8}$$

$$\tilde{y}_1(t) = y(t) - \hat{y}(t) = C\tilde{x}_1(t) + \alpha\delta(t). \tag{9}$$

Use the function $\tilde{y}_1(t)$ as information carrier about the disturbance. Write it in the following form

$$\tilde{y}_1(t) = \bar{\sigma} + \bar{\varepsilon}(t) + \sum_{i=1}^{k} \left[\bar{\mu}_i \sin(\omega_i t) + \bar{\nu}_i \cos(\omega_i t)\right]. \tag{10}$$

Multi-Sinusoidal Signal Model

It is known that the signal $\tilde{y}_1(t)$ can be generated by the following differential equation

$$p(p^2 - \theta_1)(p^2 - \theta_2)\dots(p^2 - \theta_k)\tilde{y}_1(t) = 0, \quad (11)$$

where $\theta_i = -\omega_i$, $i = 1, \dots, k$ are constant parameters.
Rewrite (11) using the Laplace transformtation

$$s^{2k+1}\tilde{y}_1(s) = \bar{\theta}_1 s^{2k-1}\tilde{y}_1(s) + \dots + \bar{\theta}_k s\tilde{y}_1(s) + d(s), \quad (12)$$

where $d(s)$ denotes initial conditions caused by the function $\varepsilon_1(t)$ and the parameters $\bar{\theta}_i$, $i = 1, \dots, k$ can be calculated by the following system

$$\begin{cases} \bar{\theta}_1 = \theta_1 + \theta_2 + \dots + \theta_k, \\ \bar{\theta}_2 = -\theta_1\theta_2 - \theta_1\theta_3 - \dots - \theta_{k-1}\theta_k, \\ \vdots \\ \bar{\theta}_k = (-1)^{k+1}\theta_1\theta_2\cdots\theta_k. \end{cases} \quad (13)$$

Regression Model

Introduce the linear filter

$$\xi(s) = \frac{\lambda_0^{2k}}{\gamma(s)}\tilde{y}_1(s), \tag{14}$$

where $\lambda_0 > 0$, $\gamma(s) = s^{2k} + \gamma_{2k-1}s^{2k-1} + \cdots + \gamma_1 s + \gamma_0$ is a Hurwitz polynomial with $2k$ different eigenvalues $\lambda_j, j = 1, \ldots, 2k$. Let $\gamma_0 = \lambda_0^{2k}$ and $\lambda = \min_{j=1,\ldots,2k}\{|\mathrm{Re}\,\lambda_j|\}$.

Multiplying (12) by $\frac{\lambda_0^{2k}}{\gamma(s)}$ with (14) we obtain

$$s^{2k+1}\xi(s) = \bar{\theta}_1 s^{2k-1}\xi(s) + \ldots + \bar{\theta}_k s\xi(s) + \frac{\lambda_0^{2k}}{\gamma(s)}d(s). \tag{15}$$

After the inverse Laplace transformation for the filter (14) and the input signal (10) we get the relation

$$\xi^{(2k+1)}(t) = \Omega^T(t)\bar{\Theta} + \varepsilon(t) \tag{16}$$

where $\Omega^T(t) = \begin{bmatrix}\xi^{(2k-1)}(t) & \ldots & \xi^{(3)}(t) & \xi^{(1)}(t)\end{bmatrix}$ is a regressor of functions $\xi^{(j)}(t)$, $\bar{\Theta}^T = \begin{bmatrix}\bar{\theta}_1 & \ldots & \bar{\theta}_{k-1} & \bar{\theta}_k\end{bmatrix}$ is a vector of unknown parameters depending on frequencies.

Adaptive Frequency Estimation Scheme

The update law

$$\hat{\omega}_i = \sqrt{\left|\hat{\theta}_i\right|}, \tag{17}$$

where estimates θ_i calculated using $\hat{\bar{\theta}}_i$ that are elements of a vector $\hat{\bar{\Theta}}$:

$$\hat{\bar{\Theta}} = \Upsilon(t) + K\Omega(t)\xi^{(2k)}(t), \tag{18}$$

$$\dot{\Upsilon}(t) = -K\Omega(t)\Omega^T(t)\hat{\bar{\Theta}}(t) - K\dot{\Omega}(t)\xi^{(2k)}(t). \tag{19}$$

where $K = \text{diag}\{k_i > 0, i = \overline{1,k}\}$, guarantees that the estimation error $\tilde{\omega}_i = \omega_i - \hat{\omega}_i$ exponentially converges to zero:

$$|\tilde{\omega}_i(t)| \le \rho_1 e^{-\beta_1 t}, \quad \rho_1, \beta_1 > 0, \quad \forall t \ge 0. \tag{20}$$

Harmonics Extracting

Write the linear filter $\xi(t)$ as follows

$$\xi(t) = \left[\frac{\lambda_0}{\gamma(p)}\right] \tilde{y}_1(t) = \left[\frac{\lambda_0}{\gamma(p)}\right] \left(\left[\frac{b(p)}{a(p)}\right] \delta(t) + \bar{\varepsilon}(t) \right) = \xi_0 + \varepsilon_2(t) + \sum_{i=1}^{k} \xi_i(t), \tag{21}$$

where

$$\xi_0 = \sigma \frac{b_0}{a_0}, \tag{22}$$

$$\begin{aligned} \xi_i(t) &= M_i \mu_i \sin(\omega_i t + \varphi_i) + M_i \nu_i \cos(\omega_i t + \varphi_i) \\ &= \begin{bmatrix} \mu_i & \nu_i \end{bmatrix} \begin{bmatrix} M_i \sin(\omega_i t + \varphi_i) \\ M_i \cos(\omega_i t + \varphi_i) \end{bmatrix} = \begin{bmatrix} \mu_i & \nu_i \end{bmatrix} \varsigma_i \end{aligned} \tag{23}$$

$$M_i = \left| \frac{\lambda_0}{\gamma(j\omega_i)} \frac{b(j\omega_i)}{a(j\omega_i)} \right|, \varphi_i = arg \left(\frac{\lambda_0}{\gamma(j\omega_i)} \frac{b(j\omega_i)}{a(j\omega_i)} \right). \tag{24}$$

For the variable $\xi(t)$ we have

$$\xi(t) = \xi_0 + \xi_1(t) + \xi_2(t) + \cdots + \xi_k(t). \tag{25}$$

We get the realizable estimation scheme for variables ξ_0 and $\xi_i(t)$:

$$\begin{bmatrix} \hat{\xi}_1(t) \\ \hat{\xi}_2(t) \\ \vdots \\ \hat{\xi}_k(t) \end{bmatrix} = \begin{bmatrix} \hat{\theta}_1 & \cdots & \hat{\theta}_k \\ \hat{\theta}_1^2 & \cdots & \hat{\theta}_k^2 \\ \vdots & \ddots & \vdots \\ \hat{\theta}_1^k & \cdots & \hat{\theta}_k^k \end{bmatrix}^{-1} \begin{bmatrix} \xi^{(2)}(t) \\ \xi^{(4)}(t) \\ \vdots \\ \xi^{(2k)}(t) \end{bmatrix}, \quad \hat{\xi}_0(t) = \xi(t) - \sum_{i=1}^{k} \hat{\xi}_i(t). \tag{26}$$

Amplitudes and Bias Identification

The bias can be found as follows

$$\hat{\sigma} = \frac{a_0}{b_0}\hat{\xi}_0. \tag{27}$$

Introducing the auxiliary function of time $\hat{\Delta}(t) = \sum_{i=0}^{k} \sin(\hat{\omega}_i t)$ consider the auxiliary filter

$$\vartheta(t) = \left[\frac{\lambda_0}{\gamma(p)}\frac{b(p)}{a(p)}\right]\hat{\Delta}(t) = \sum_{i=1}^{k}\vartheta_i(t) + \varepsilon_3(t). \tag{28}$$

The signals $\vartheta_i(t)$ and its derivatives equal

$$\vartheta_i(t) = M_i \sin(\omega_i t + \varphi_i), \quad \dot{\vartheta}_i(t) = \omega_i\, M_i \cos(\omega_i t + \varphi_i). \tag{29}$$

Using the algorithm of harmonic extraction design the observer for all components of regressors ς_i

$$\begin{bmatrix} \hat{\vartheta}_1(t) \\ \hat{\vartheta}_2(t) \\ \vdots \\ \hat{\vartheta}_k(t) \end{bmatrix} = \begin{bmatrix} \hat{\theta}_1 & \cdots & \hat{\theta}_k \\ \hat{\theta}_1^2 & \cdots & \hat{\theta}_k^2 \\ \vdots & \ddots & \vdots \\ \hat{\theta}_1^k & \cdots & \hat{\theta}_k^k \end{bmatrix}^{-1} \begin{bmatrix} \vartheta^{(2)}(t) \\ \vartheta^{(4)}(t) \\ \vdots \\ \vartheta^{(2k)}(t) \end{bmatrix}, \tag{30}$$

$$\begin{bmatrix} \hat{\dot{\vartheta}}_1(t) \\ \hat{\dot{\vartheta}}_2(t) \\ \vdots \\ \hat{\dot{\vartheta}}_k(t) \end{bmatrix} = \begin{bmatrix} 1 & \cdots & 1 \\ \hat{\theta}_1 & \cdots & \hat{\theta}_k \\ \vdots & \ddots & \vdots \\ \hat{\theta}_1^{k-1} & \cdots & \hat{\theta}_k^{k-1} \end{bmatrix}^{-1} \begin{bmatrix} \vartheta^{(1)}(t) \\ \vartheta^{(3)}(t) \\ \vdots \\ \vartheta^{(2k-1)}(t) \end{bmatrix}. \tag{31}$$

Amplitudes and Bias Identification

So we get the algorithm to calculate the regressor functions $\varsigma_i(t)$

$$\hat{\varsigma}_i^T(t) = \begin{bmatrix} \hat{\vartheta}_i(t) & \frac{\hat{\dot{\vartheta}}_i(t)}{\eta_i(t)} \end{bmatrix}, \tag{32}$$

$$\eta_i(t) = \begin{cases} \hat{\omega}_i(t), & \text{if } \hat{\omega}_i \geq \omega_0, \\ \omega_0, & \text{otherwise}, \end{cases} \tag{33}$$

Eventually, we design the update laws for μ_i and ν_i by the standard approach basing on the regressor model (23)

$$\dot{\hat{\mu}}_i(t) = k_\mu \, \hat{\vartheta}_i(t) \left(\hat{\xi}_i(t) - \hat{\mu}_i(t)\hat{\vartheta}_i(t) - \hat{\nu}_i(t) \frac{\hat{\dot{\vartheta}}_i(t)}{\eta_i(t)} \right), \tag{34}$$

$$\dot{\hat{\nu}}_i(t) = k_\nu \frac{\hat{\dot{\vartheta}}_i(t)}{\eta_i(t)} \left(\hat{\xi}_i(t) - \hat{\mu}_i(t)\hat{\vartheta}_i(t) - \hat{\nu}_i(t) \frac{\hat{\dot{\vartheta}}_i(t)}{\eta_i(t)} \right), \tag{35}$$

where $k_\mu, k_\nu > 0$ and $\eta_i(t)$ is defined above in (33) .

Resultant Disturbance Observer

Using $\hat{\sigma}$, $\hat{\mu}_i$ и $\hat{\nu}_i$ it is easy to obtain disturbance observer $\delta(t)$

$$\hat{\delta}(t) = \hat{\sigma} + \sum_{i=1}^{k} \left[\hat{\mu}_i \sin(\hat{\omega}_i t) + \hat{\nu}_i \cos(\hat{\omega}_i t)\right]. \qquad (36)$$

The second observer taking into account the known delay D we can get as follows

$$\begin{aligned} \hat{\delta}(t+D) &= \hat{\sigma} + \sum_{i=1}^{k} \left[\hat{\mu}_i \sin(\hat{\omega}_i (t+D)) + \hat{\nu}_i \cos(\hat{\omega}_i (t+D))\right] \qquad (37) \\ &= \hat{\sigma} + \sum_{i=1}^{k} \left[\kappa_i \sin(\hat{\omega}_i t) + \zeta_i \cos(\hat{\omega}_i t)\right], \end{aligned}$$

where

$$\kappa_i = \hat{\mu}_i \cos(\hat{\omega}_i D) - \hat{\nu}_i \sin(\hat{\omega}_i D), \qquad (38)$$

$$\zeta_i = \hat{\mu}_i \sin(\hat{\omega}_i D) + \hat{\nu}_i \cos(\hat{\omega}_i D). \qquad (39)$$

Control Design

The feedback will be chosen in the form

$$u(t) = -\hat{\delta}(t + D) + \psi(t). \tag{40}$$

Substituting (40) into (1) yields

$$\dot{x}(t) = Ax(t) + B\psi(t - D) + B\tilde{\delta}_1(t). \tag{41}$$

Basing on (41) we introduce the second observer for state variables.

$$\dot{\hat{x}}_2(t) = A\hat{x}_2 + B\psi(t - D) + L\tilde{y}_2(t), \tag{42}$$

$$\hat{y}_2(t) = C\hat{x}_2(t) + \alpha\hat{\delta}(t), \tag{43}$$

Consider a model of the observer error $\tilde{x}_2(t) = x(t) - \hat{x}(t)$ with respect to (1), (2), (42), (43)

$$\dot{\tilde{x}}_2(t) = (A - LC)\tilde{x}_2(t) + B\tilde{\delta}_1(t) - L\alpha\tilde{\delta}_2(t), \tag{44}$$

$$\tilde{y}_2(t) = C\tilde{x}_2(t) + \alpha\tilde{\delta}_2(t). \tag{45}$$

Control Design

Consider model of the delay by the transport PDE (M. Krstic approach)

$$\Psi_t(z,t) = \Psi_z(z,t), \quad 0 < z < D \tag{46}$$

$$\Psi(D,t) = \psi(t) \tag{47}$$

with the initial condition $\Psi(z,0) = \psi(z-D)$. The solution of this PDE is $\Psi(z,t) = \psi(t+z-D)$, and therefore $\Psi(0,t) = \psi(t-D)$ gives the delayed input. We can now rewrite (41) in the form

$$\dot{x}(t) = Ax(t) + B\Psi(0,t) + B\tilde{\delta}_1(t). \tag{48}$$

Choose the control law for $\psi(t)$ in the form

$$\psi(t) = Ke^{AD}\hat{x}_2(t) + K\int_0^D e^{A(D-\tau)}B\Psi(\tau,t)d\tau\,. \tag{49}$$

Numerical Example

Consider the following linear plant

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0,1 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(t-1) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} \delta(t), \tag{50}$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) + \delta(t), \tag{51}$$

where the disturbance signal has a view

$$\delta(t) = 1 + 3\sin(0,8\ t) - \cos(0,8\ t). \tag{52}$$

Numerical Example

Introduce the first state observer to extract the information about the disturbance from $y(t)$

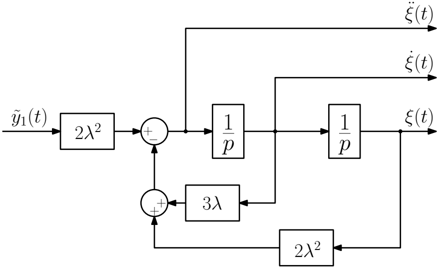
$$\dot{\hat{x}}_1(t) = \begin{bmatrix} 0 & 1 \\ 0,1 & -1 \end{bmatrix} \hat{x}_1(t) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(t-1) + L\left(y(t) - \hat{y}_1(t)\right), \quad (53)$$

$$\hat{y}_1(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \hat{x}_1(t), \quad (54)$$

where choose the matrix $L^T = \begin{bmatrix} 2 & 2 \end{bmatrix}$.
The error signal $\tilde{y}_1(t) = y(t) - \hat{y}_1(t)$ will be used as information carrier about the disturbance.
Introduce the linear filter

$$\xi(t) = \left[\frac{2\lambda^2}{(p^2 + 3\lambda p + 2\lambda^2)}\right] \tilde{y}_1(t), \quad (55)$$

where $\lambda = 1$.



Numerical Example

Write the realizable scheme of frequency identification

$$\hat{\omega}(t) = \sqrt{\left|\hat{\theta}(t)\right|}, \tag{56}$$

$$\hat{\theta}(t) = \chi(t) + k\dot{\xi}(t)\ddot{\xi}(t), \tag{57}$$

$$\dot{\chi}(t) = -k\dot{\xi}^2(t)\hat{\theta}(t) - k\ddot{\xi}^2(t), \tag{58}$$

where $k = 2$.
To avoid division by zero introduce the following constraint for the frequency

$$\hat{\omega}(t) = \omega_0, \quad \text{if } \hat{\omega} < \omega_0, \tag{59}$$

where $\omega_0 = 0.5$.
Write the equations for the components of the signal $\xi(t)$

$$\hat{\xi}_1(t) = -\frac{\ddot{\xi}(t)}{\hat{\omega}^2(t)}, \qquad \hat{\xi}_0(t) = \xi(t) - \hat{\xi}_1(t). \tag{60}$$

Numerical Example

The bias can be found as

$$\hat{\sigma}(t) = \frac{a_0}{b_0}\hat{\xi}_0(t), \tag{61}$$

where $b_0 = 1.9$ и $a_0 = 3.9$ correspond to the transfer function

$$\frac{b(p)}{a(p)} = C(pI - (A - LC))^{-1}(B - L\alpha) + \alpha = \frac{p^2 + p + 1.9}{p^2 + 3p + 3.9}. \tag{62}$$

Taking into consideration the auxiliary variable $\hat{\Delta}(t) = \sin(\hat{\omega}t)$ consider the filter for $\hat{\Delta}(t)$

$$\hat{\vartheta}(t) = \left[\frac{2(p^2 + p + 1,9)}{(p^2 + 3p + 2)(p^2 + 3p + 3,9)}\right]\hat{\Delta}(t). \tag{63}$$

The algorithm for computation of $\hat{\mu}$ and $\hat{\nu}$ looks as follows

$$\dot{\hat{\mu}}(t) = k_\mu\,\hat{\vartheta}(t)\left(\hat{\xi}_1(t) - \hat{\mu}(t)\hat{\vartheta}(t) + \hat{\nu}(t)\frac{\dot{\hat{\vartheta}}(t)}{\hat{\omega}^2(t)}\right), \tag{64}$$

$$\dot{\hat{\nu}}(t) = -k_\nu\,\frac{\dot{\hat{\vartheta}}(t)}{\hat{\omega}^2(t)}\left(\hat{\xi}_1(t) - \hat{\mu}(t)\hat{\vartheta}(t) + \hat{\nu}(t)\frac{\dot{\hat{\vartheta}}(t)}{\hat{\omega}^2(t)}\right), \tag{65}$$

where $k_\mu = 2$ and $k_\nu = 3$.

Numerical Example

The first disturbance observer (without delay) has the form

$$\hat{\delta}(t) = \hat{\sigma} + \hat{\mu}\sin(\hat{\omega}t) + \hat{\nu}\cos(\hat{\omega}t). \tag{66}$$

The second disturbance observer (with delay) has the form

$$\hat{\delta}(t+D) = \hat{\sigma} + \kappa\sin(\hat{\omega}t) + \zeta\cos(\hat{\omega}t), \tag{67}$$

$$\kappa = \hat{\mu}\cos(\hat{\omega}D) - \hat{\nu}\sin(\hat{\omega}D), \tag{68}$$

$$\zeta = \hat{\mu}\sin(\hat{\omega}D) + \hat{\nu}\cos(\hat{\omega}D). \tag{69}$$

Write the second state observer for main control scheme

$$\dot{\hat{x}}_2(t) = A\hat{x}_2 + B\psi(t-D) + L\left(y(t) - \hat{y}_2(t)\right), \tag{70}$$

$$\hat{y}_2(t) = C\hat{x}_2(t) + \alpha\hat{\delta}(t). \tag{71}$$

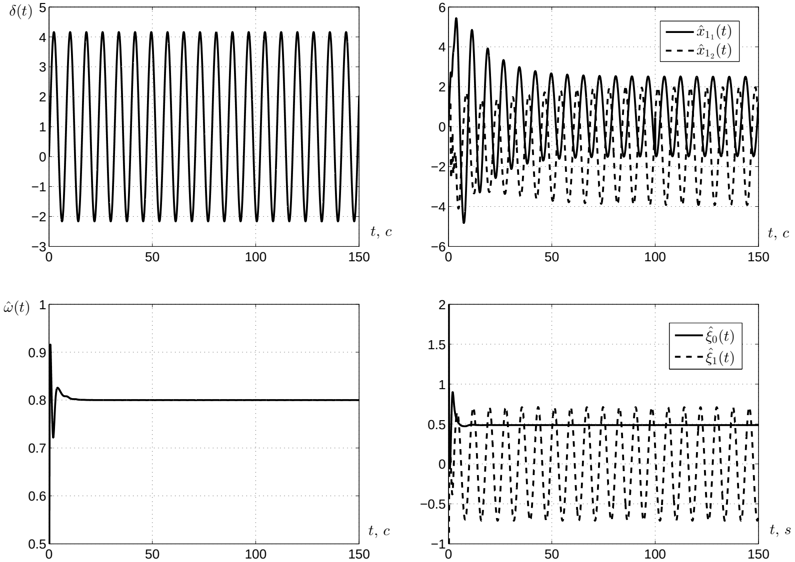
Choose the control signal

$$u(t) = -\hat{\delta}(t+D) + \psi(t), \tag{72}$$

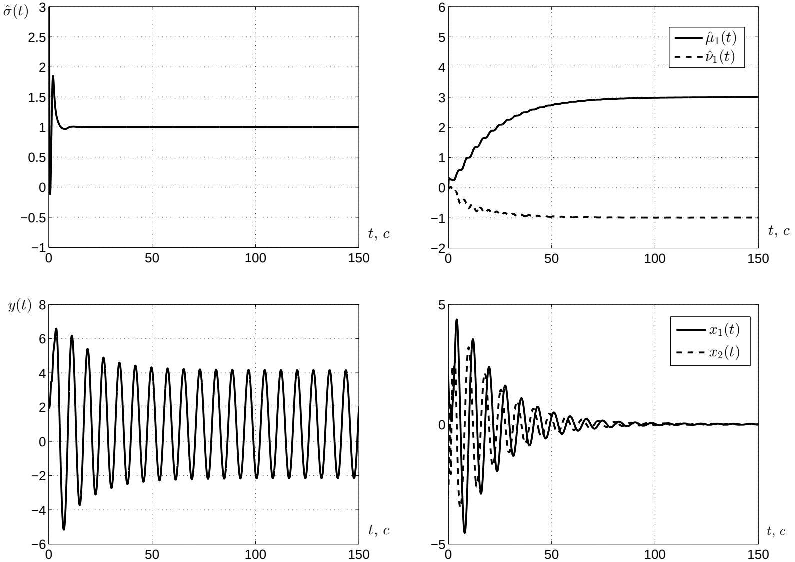
$$\psi(t) = Ke^{AD}\hat{x}_2(t) + K\int_{t-D}^{t} e^{A(t-z)}B\psi(z)dz\,, \tag{73}$$

where $K = [2 \quad 2]$.

Numerical Example



Numerical Example



Conclusions

- the adaptive controller for the linear plant with the input delay was designed
- the obtained algorithm cancels the unknown disturbance affecting on the input and the output providing exponentially decaying to zero for the state variables
- the effectiveness of the proposed approach was illustrated by the numerical example