Equilibrium transportation model with developers

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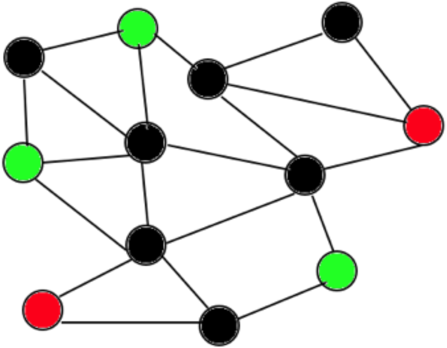
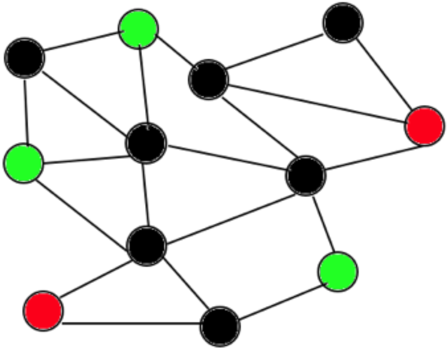
Given graph weighted graph $\Gamma(V, E)$
Capacity vector $\bar{f}$
Free flow travel time vector $\bar{\tau}$
OD-matrix $W = \{d_{ij}\}$



$C_p(\tau)$ — travel cost for directed path p under travel cost vector τ.
P_{ij} — set of available paths for od-pair (i,j).
We call (f^{eq}, τ^{eq}) equilibrium iff:
$\forall (i,j) \in W$
$\forall p \in P_{ij}$ s.t. $f_e^{eq} > 0 \ \forall e \in p$
holds
$C_p(\tau^{eq}) = min_{q \in P_{ij}} C_q(\tau^{eq})$
and
$\forall p \in P_{ij}$ s.t. $\exists e \in p$, $f_e^{eq} = 0$
holds
$C_p(\tau^{eq}) \geq min_{q \in P_{ij}} C_q(\tau^{eq})$

Equilibrium is the solution of the following optimization problem

$$\max_{\overline{\tau}\leq\tau}\{\sum_{i,j=1}^{n,n} T_{ij}(\tau)\cdot d_{ij} - \langle\overline{f},\tau-\overline{\tau}\rangle\}.$$



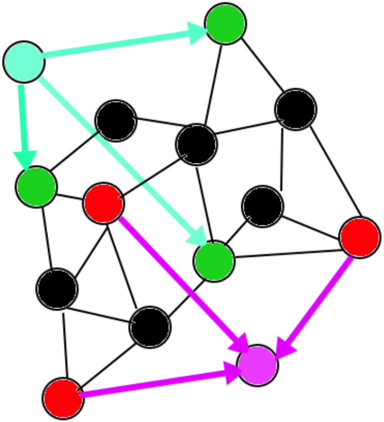
Usually we don't know OD-matrix W.
But we can estimate

$$L_i = \sum_j d_{ij}$$

$$W_j = \sum_i d_{ij}$$

$$\sum_i L_i = \sum_j W_j = N$$

Add false nodes s and t and connect them with origin and destination nodes respectively.



Now we can get equilibrium as the solution of the following problem.

$$\max_{\tau_e, e \notin E} \max_{\tau_e \geq \overline{\tau}_e, e \in E} NT_{st}(\tau) - \sum_{e \in E} (f_e, \tau_e - \overline{\tau}_e) - \sum_{i \in V} L_i \cdot \tau_{it} - \sum_{i \in V} W_i \cdot \tau_{it} \quad (1)$$

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Question: Who choose $\bar{f}$ and $\bar{\tau}$?
Answer: Those guys, who build roads and buildings!

New agents: developers
Set of strategies: choose available $\bar{f}$ and $\bar{\tau}$
Payoffs:
$\bar{\tau}_e \cdot f_e^{eq} - c_e \cdot \bar{f}_e$ for selfish developer
$-\bar{\tau}_e \cdot f_e^{eq} - c_e \cdot \bar{f}_e$ for social oriented developer (goverment)
Here c_e is the cost of constraction, f_e^{eq} equilibrium flow on arc e.

Theorem: Joint equilibrium could be found as the solution of the following optimization problem:

$$\max_{\bar{f}_e, \bar{\tau}_e, e \in E_1} \min_{\bar{f}_e, \bar{\tau}_e, e \in E_2} \min_{\bar{f}_e, e \in E} \min_{f} \sum_{e \in \hat{E}} f_e \cdot \bar{\tau}_e + \sum_{e \in E} \bar{f}_e \cdot c_e + \sum_{e=(i,t), i \in D} \bar{f}_e \cdot c_e - \sum_{e=(s,i), i \in O} \bar{f}_e \cdot \quad (2)$$

$$\mu \leq \bar{f} \leq \gamma$$

$$0 \leq f \leq \bar{f}$$

$$\sum_{i \in O} f_{(s,i)} = N$$

$$\sum_{i \in D} f_{(i,t)} = N$$

$$\sum_{i:(i,v) \in E} f_{(i,v)} - \sum_{i:(v,i) \in E} f_{(v,i)} = 0$$

Thanks for your attantion!