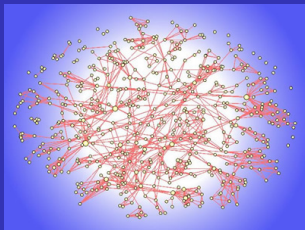


# Consensus and polarization of opinions in social groups: a simple model



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VI Traditional School, Grigorchikovo, June 2014

# Complex systems: a new philosophy of sciences

Instead of definitions

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## Physical methods

Dynamics of many systems is so complex that only physical methods (coming from statistical physics/physics of disordered matter are applicable). This idea gave birth to such branches as econophysics, sociophysics, macroscopic traffic flow models etc.

# Another approach – networked/multi-agent systems

Some words about concepts

## IFAC TC 1.5 (Hideaki Ishii)

Networked systems are complex dynamical systems composed of a large number of simple systems interacting through a communication medium (or otherwise). These systems arise as natural models in many areas ... such as sensor networks, autonomous unmanned vehicles, biological networks, and animal cooperative aggregation and flocking.

## MAS (Wikipedia)

A multi-agent system (M.A.S.) is a ... system composed of multiple interacting intelligent *agents* within an environment. The agents in a multi-agent system have several important characteristics:

- *Autonomy*: the agents are at least partially independent, self-aware, autonomous;
- *Local views*: no agent has a full global view of the system, or the system is too complex for an agent to make practical use of such knowledge;
- *Decentralization*: there is no designated controlling agent;



# Multi-agent Systems (MAS)

Control theory

## MAS vs Classical (centralized) control

- local interactions, coupling between “neighboring agents”;
- local control/decisions (intellectual agents);
- complex and time-varying interaction topology;
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The difference (as compared with classical control) is well elucidated by the inscription on the Ring (J. Tolkien)

*One Ring to rule them all, One Ring to find them.*

*One Ring to bring them all and in the darkness bind them*

*In the Land of Mordor where the Shadows lie.*

and especially in one of translations to Russian.

# Multi-agent Systems (MAS)

## Other applications

- Biology, e.g. models of biological formations (flocks, herds, schools);
- Physics, e.g. Vicsek models (self-propelled particles);
- Transport/logistics, e.g. microscopic models of traffic flow;
- **Social/behavioral studies**, e.g. spread of “memes” in social networks (opinions, epidemic diseases, social power etc.);

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## The problem in question

Opinion dynamics in large-scale social networks. The aim is to explain how consensus, polarization, clusterization of opinions. We focus on very simple multi-agent models.

# Some assumptions and notations

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- The interaction between two agents is determined by the **coupling map**  $\varphi_{jk} : \mathbb{R}^d \times \mathbb{R}^d \rightarrow \mathbb{R}^d$  (corresponds to the “dyadic tie”).
- Resulting model of network may be discrete-time

$$x_j(t+1) = \sum_{k=1}^N a_{jk}(t) \varphi_{jk}(x_k(t), x_j(t))$$

or continuous-time

$$\dot{x}_j(t) = \sum_{k=1}^N a_{jk}(t) \varphi_{jk}(x_k(t), x_j(t)).$$

# Origins: De Groot Model

Consensus in Expert Communities

## Original informal setup

Consider a random element  $\theta \in \{1, 2, \dots, M\}$  with unknown distribution  $p_* = (p_1, \dots, p_M)^T$ . A group of  $N$  experts wants to estimate  $p_*$  by iterative tuning of their own opinions  $p^j = (p_1^j, \dots, p_M^j)^T$ . What is a “natural” consensus estimate  $\hat{p}_*$  and how to reach it.

## Game-theoretic approach (pari-mutuel model)

Let  $\theta$  is a number of horse which is going to win the race. Render the agents to make bets proportionally to their opinions. Consensus is the Nash equilibrium (Eisenberg, Gale, 1958). *Problems*: poor convergence properties, focused on probability distributions.

Another concept: **averaging** (Winkler, 1968; De Groot, 1971) Each agent assigns weights to herself and her neighbors  $a_{jk}(s) \geq 0$ ,  $\sum_{k=1}^N a_{jk} = 1$ . Consensus means:

$$p^j = \sum_{k=1}^N a_{jk} p^k \Leftrightarrow P := [p^1, p^2, \dots, p^N] = AP.$$

# Origins: De Groot Model

Concept of "Iterative Pooling"

## Algorithm for reaching consensus (De Groot, 1971)

A very simple iterative procedure

$$p(t+1) = Ap(t) \in \mathbb{R}^N, \quad A \text{ is stochastic.}$$

More general situation:  $A = A(t)$  (weights are evolving). Consensus:  $p(t) \rightarrow c(1, 1, \dots, 1)^T$  as  $t \rightarrow \infty$ .

## Well developed mathematical techniques

- Consensus=convergence of infinite matrix products;
- If  $A(t) \equiv A$  then consensus $\Leftrightarrow$   $A$  is SIA;
- More general results: Wolfowitz's theorem etc. See the survey by Agaev and Chebotarev, UBS, 2010, 30.1 (in Russian).

# Continuous-time models

Agaev and Chebotarev, 1999; Olfati-Saber and Murray, 2004; Ren and Beard, 2005

Update time is small relatively to the system life

$$\dot{x}_j(t) = \sum_{k=1}^N a_{jk}(t)(x_k(t) - x_j(t)), \quad a_{jk} \geq 0 \Leftrightarrow \dot{x}(t) = -L(t)x(t).$$

Properties are almost exhaustively investigated.

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- If  $A(t) \equiv A$  then consensus  $\Leftrightarrow$  quasi-strong connectivity (existence of oriented spanning tree);
- Necessary condition: let  $b_{jk}(t) = 1$  if  $\int_0^\infty a_{jk}(s)ds = \infty$  and  $b_{jk} = 0$  otherwise. Then  $B$  defines a QSC graph;



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- In general one needs more restrictive condition of *uniform connectivity*. Let  $b_{jk}(t) = 1$  if  $\int_0^T a_{jk}(s)ds > \varepsilon$  and  $b_{jk} = 0$  otherwise. Then  $B$  defines a QSC graph.

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- Proofs are nonlinear and based on **shrinking** of  $\text{conv}\{x_1, \dots, x_N\}$ .

# Beyond consensus: agents failed to agree

More typical situation

## Mechanism underlying consensus:

A closer analysis of the consensus scheme reveals assumptions:

- connectivity;
- flexibility of agents (opinions are evolving);
- structure of couplings (averaging);
- cooperation (agents desire to make their opinions closer);

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## Why consensus may be broken?

- disconnected topology;
- stubborn agents (having their **intrinsic** opinion);
- general nonlinear couplings;
- competition, or **negative ties** (an agent dislikes opinions of her *foes*);

# Beyond consensus: agents failed to agree

Some conventional models

Stubborn agents, e.g. DeGroot-Friedkin-Johnssen model

$$x(t+1) = A(t)x(t) + B(t)c, \quad A(t) + B(t) \text{ is stochastic}$$

If  $c_j \neq c_k$ , may lead to clustering of opinions (e.g. if  $A \equiv 0$ ).

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## Homophily-based models

Homophily (i.e., "love of the same") is the tendency of individuals to associate and bond with similar others.

- Bounded confidence models (Hegselmann, Krause, 2002). The agents interact only to like-minded ones;

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- Biased assimilation (Dandekar, Goel, Lee, 2013). Special nonlinear couplings, attracting the agents to their own opinions.



# Beyond consensus: agents failed to agree

A new simple model

Altafini's model: explains polarization (PLoS One, 2012; IEEE TAC 2013)

Idea: introduce **negative ties**, or **repulsive couplings**. The matrix of weights  $A = (a_{jk})$  is *signed*.

$$\dot{x}_j(t) = \sum_{k=1}^N |a_{jk}(t)| (x_k(t) \text{sign } a_{jk}(t) - x_j(t)), \quad \dot{x}(t) = -L(t)x(t).$$

It can be shown that  $|x(t)|_\infty$  is non-increasing, so  $x_j(0) \in [-M; M] \forall j$  entails  $|x_j(t)| \leq M$ . The set of admissible opinions can not be arbitrary.

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Assume the graph constant and **strongly** connected. Then

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- if the graph is *structurally balanced*, opinions polarize;
- idea of the proof is based on *gauge transformation*, extends to nonlinear case;

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A. Proskurnikov, A. Matveev and M.Cao (2014, to be submitted)

## The case of static graph ( $A=\text{const}$ )

The assumption of strong connectivity may be removed.

- steady opinions polarize (and non-zero) if and only if the graph is structurally balanced and QSC;
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- In general one needs more restrictive condition of *uniform strong connectivity*. Let  $b_{jk}(t) = 1$  if  $\int_0^T |a_{jk}(s)| ds > \varepsilon$  and  $b_{jk} = 0$  otherwise. If  $B$  defines a QSC graph for some  $\varepsilon, T$ , then modulus consensus is achieved.



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- Nonlinear couplings may be considered as well.

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## Important Dates

- **October 1st, 2014**  
Paper submission
- **December 20th, 2014**  
Submission deadline
- **February 20th, 2015**  
Notification of acceptance
- **April 15th, 2015**  
Final paper submission deadline
- **June 24-26th, 2015**  
Conference dates

**MICNON 2015** is going to be the first of a new conference series that is organized by the IFAC Technical Committee on Nonlinear Systems. The scope of the conference will cover all areas of nonlinear systems theory and applications in science and engineering, including control of nonlinear systems, analysis of nonlinear systems, modelling and identification of nonlinear systems and all types of applications in connection to nonlinear systems. English will be the official language of the Workshop. No simultaneous translation will be provided.

**Meeting Topics include but are not limited to:**

- modeling and identification of nonlinear systems;
- control of nonlinear systems;
- stability and complex dynamics;
- networked nonlinear systems;
- stochastic control systems;
- control of networks;
- control with limited information;
- nonlinear systems with time delay;
- disturbance rejection;
- switching control;
- adaptive control and signal processing for nonlinear systems;
- other related topics.

**IFAC Technical Committee Main-Sponsor:**

TC 2.3 Non-Linear Control Systems

**IFAC Technical Committee(s) Co-Sponsors:**

TC 1.1 Modeling, Identification and Signal Processing

TC 1.2 Adaptive and Learning Systems

TC 1.5 Networked Systems

TC 2.4 Optimal control

TC 6.1 Chemical Process Control

TC 9.4. Control Education

**Conference secretariat:**

**A.V. Proskurnikov**

**Consensus and polarization of opinions: a simple model**

# THANK YOU FOR YOUR ATTENTION

Questions?