Passification-based adaptive control with quantized measurements

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Passification-based adaptive control

Consider an uncertain linear system

$$\begin{aligned} \dot{x}(t) &= A_\xi x(t) + B_\xi u(t), \\ y(t) &= C_\xi x(t), \end{aligned} \qquad x \in \mathbb{R}^n, y \in \mathbb{R}^l, u \in \mathbb{R}, \xi \in \Xi.$$

Assumption

$\exists g \in \mathbb{R}^l : \forall \xi \in \Xi \quad g^T C_\xi (sI - A_\xi)^{-1} B_\xi$ *is hyper-minimum-phase, that is* $g^T W_\xi(s) \det(sI - A_\xi)$ *is stable with* $g^T C_\xi B_\xi > 0$.

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Adaptive control:

$$u(t) = -k(t) g^T y(t),$$
$$\dot{k}(t) = \gamma \left(g^T y(t)\right)^2,$$

where $\gamma > 0$.
The closed-loop system is such that[1] $x(t) \to 0$, $k(t) \to \text{const}$.

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Quantizer

Quantizer[2]:

$$q\colon \mathbb{R}^l \to Q,$$

where Q is a finite subset of $\mathbb{R}^l$. We assume there exist $M > \Delta > 0$:

$$\|y\| \leq M \Rightarrow \|q(y) - y\| \leq \Delta.$$

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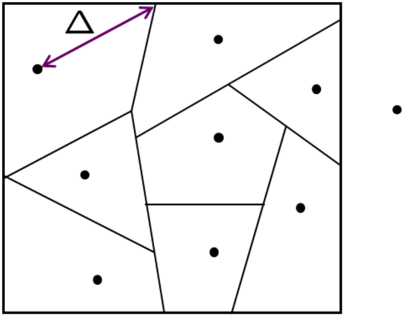

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$$q_\mu(y) = \mu q\left(\frac{y}{\mu}\right).$$

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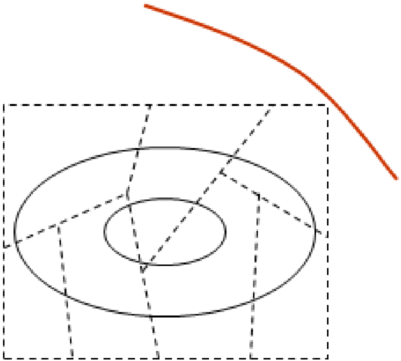

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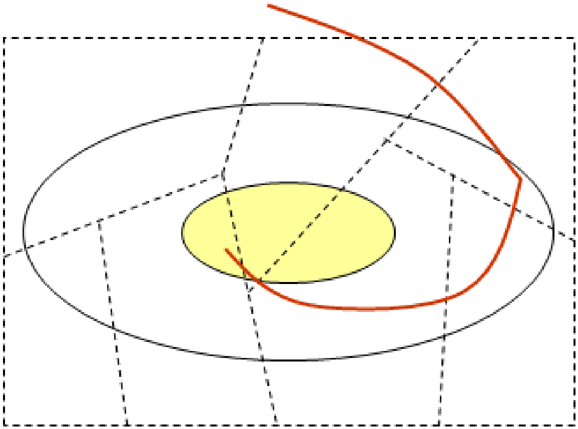

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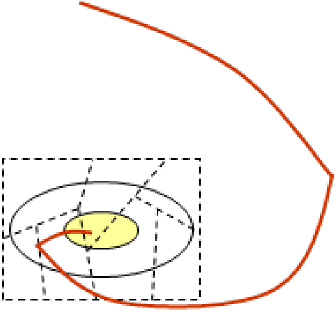

Adaptive controller with quantized measurements

We consider an uncertain system

$$\begin{aligned}\dot{x}(t) &= A_\xi x(t) + B_\xi u(t),\\ y(t) &= C_\xi x(t),\end{aligned} \qquad x \in \mathbb{R}^n, y \in \mathbb{R}^l, u \in \mathbb{R}, \xi \in \Xi, \tag{1}$$

under the adaptive controller

$$\begin{aligned}u(t) &= -k(t) g^T q_{\mu(t)}(y(t)),\\ \dot{k}(t) &= \gamma [g^T q_{\mu(t)}(y(t))]^2 - a(t)k(t),\end{aligned} \tag{2}$$

where $\gamma > 0$; $a(t)$, $\mu(t)$ – piecewise constant functions; $q_{\mu(t)}$ dynamic quantizer.

Basic idea. Passification lemma.

If $g^T C_\xi (sI - A_\xi)^{-1} B_\xi$ is HMP there exist[3] P_ξ, $\varkappa_*$ such that

$$P_\xi > 0, \quad P_\xi A_\varkappa + A_\varkappa^T P_\xi < 0, \quad P_\xi B_\xi = C_\xi^T g,$$

where $A_\varkappa = A_\xi - B_\xi \varkappa_* g^T C_\xi$.

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Denote

$$k_* = \varkappa_* + \frac{\varkappa_*}{\sqrt{2}}$$

and consider Lyapunov-like function

$$V(x, k) = x^T P_\xi x + \gamma^{-1}(k - k_*)^2.$$

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Assume that we know some V_0 such that

$$V(x(0), k(0)) \leq V_0.$$

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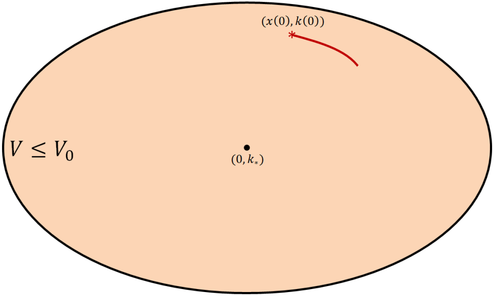
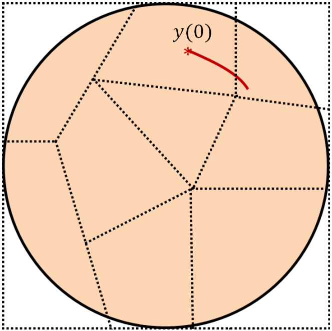
Basic idea. Converging ellipsoids.

$$V(x,k) = x^T P_\xi x + \gamma^{-1}(k - k_*)^2$$

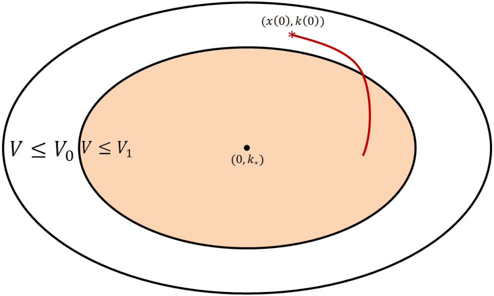


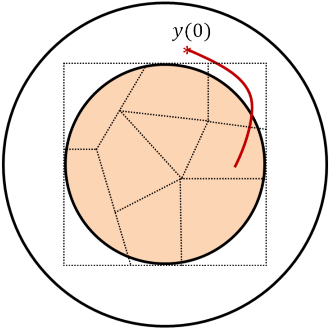
$a(t) = a_0$



$\mu(t) = 1$

$$V(x,k) = x^T P_\xi x + \gamma^{-1}(k - k_*)^2$$





$$a(t) = a_1$$

$$\rho_2 \Delta^2 < V_0 \Rightarrow V_1 < V_0$$

$$\mu(t) = \sqrt{V_1 V_0^{-1}}$$

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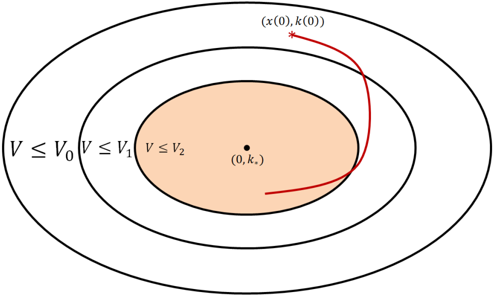
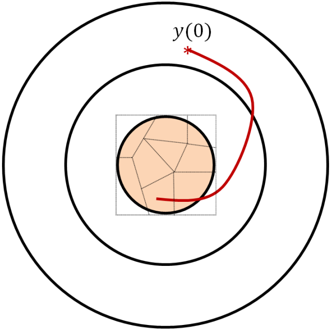


$a(t) = a_2$



$\mu(t) = \sqrt{V_2 V_0^{-1}}$

Limit ellipsoid

Ellipsoids will converge to

$$V(x,k) \leq V_\infty = \frac{\rho_1}{1 - \rho_2 \Delta^2 V_0^{-1}},$$

where

$$\rho_1 = \left(1 + \frac{1}{\sqrt{2}}\right)^2 \frac{\varkappa_*^2}{\gamma}, \quad \rho_2 = \left(3 + 2\sqrt{2}\right) \frac{2\varkappa_*}{\alpha}.$$

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$$t_i \to t_* < \infty \text{ for } i \to \infty.$$

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Solution: we turn off switching when $V \leq V_\heartsuit$, where $V_\heartsuit$ is the desired value of V.

Switching procedure

Let us fix some $\alpha \in (0, -2\lambda)$, where

$$\lambda = \max\{\mathrm{Re}(s) | g^T W_\xi(s) = 0,\ \xi \in \Xi\}.$$

Estimates of V:

$$V_{i+1} = \rho_1 + \rho_2 V_i V_0^{-1} \Delta^2 + \nu. \tag{3}$$

Switching instants:

$$t_{i+1} = t_i + \frac{1}{\alpha} \ln \frac{V_i - \rho_1 - \rho_2 \frac{V_i}{V_0} \Delta^2}{\nu}, \quad t_0 = 0. \tag{4}$$

Zooming:

$$\mu(t) = \mu_i = \sqrt{V_i V_0^{-1}}, \quad t \in [t_i, t_{i+1}). \tag{5}$$

Regularization parameter:

$$a(t) = a_i = \alpha + \gamma \mu_i^2 \Delta^2 \left(\frac{1}{\sqrt{\gamma \rho_1}} + \frac{\sqrt{2}}{\varkappa_*} \right), \quad t \in [t_i, t_{i+1}). \tag{6}$$

Main result

Theorem

Suppose that $g^T C_\xi(sI - A_\xi)B_\xi$ *is HMP and* $\rho_2\Delta^2 < V_0$. *Then for any* $\varepsilon > 0$ *there exist* $\gamma > 0$ *and* $\nu > 0$ *such that the controller* (2) *with the switching algorithm* (3)-(6) *ensures existence of such* i *that*

$$\forall t \geq t_i \quad \|x(t)\| < \varepsilon.$$

Moreover, the tuning coefficient $k(t)$ *is bounded for* $t \geq 0$.

Example: yaw angle control

Lateral motion of an aircraft[4]

$$\begin{cases} \dot{\beta}(t) = r(t) + c_1\beta(t) + b_1 u(t), \\ \dot{r}(t) = c_2\beta(t) + c_3 r(t) + b_2 u(t), \\ \dot{\psi}(t) = r(t), \end{cases} \quad y(t) = \begin{pmatrix} r(t) \\ \psi(t) \end{pmatrix}.$$

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For $g = \frac{1}{\sqrt{2}}(1,1)^T$ the transfer function has the form

$$g^T W(s) = \frac{b_2 s^2 + (b_1 c_2 - b_2 c_1 + b_2)s + b_1 c_2 - b_2 c_1}{s\sqrt{2}(s^2 - (c_1 + c_3)s + c_1 c_3 - c_2)}.$$

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We took the following parameters: $c_1 \in [0.1, 1.5]$, $c_2 \in [25, 40]$, $c_3 = 1.3$, $b_1 = 19/15$, $b_2 = 19$.

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Numerical simulations

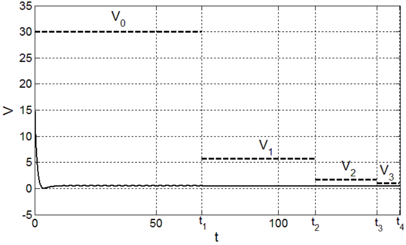

Figure 1: Evolution of $V(x(t), k(t))$. $V_\infty \approx 0.9268$, $V_3 < V_\infty + 0.2$.

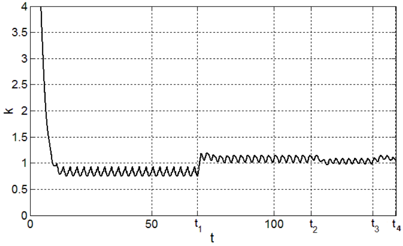

Figure 2: Evolution of $k(t)$

Numerical simulations

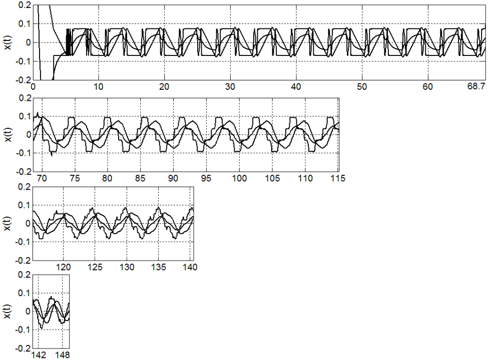

Figure 3: Evolution of the state

i	t_i	V_i	μ_i	a_i
0	0	30	1	0.374
1	68.7	5.73	0.437	0.136
2	115.15	1.72	0.239	0.097
3	140.53	1.06	0.188	0.090
4	149.76	0.95	0.178	0.089

Figure 4: Parameters of switching

Summary:
For hyper-minimum-phase linear uncertain systems with quantized measurements we derived an adaptive controller with switching zooming and regularization parameters that insures convergence of a state to a given set.